Performance characterization of chirped return-to-zero modulation format using an accurate receiver model

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Abstract

We derive an explicit relationship between the Q-factor and the optical signal-to-noise ratio in optical fiber transmission systems for an arbitrary pulse shape using an accurate receiver model under the assumption that the noise is unpolarized. We also define the enhancement factor and two other parameters that explicitly quantify the relative performance of different modulation formats. We use this method to compare the performance of the chirped return-to-zero, return-to-zero, and nonreturn-to-zero modulation formats with a finite extinction ratio. The method that we propose can also be used as a tool for the design and optimization of optical receivers.

Index Terms

Optical communications, modulation format, Q-factor, optical signal-to-noise ratio, bit-error-rate.

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I. INTRODUCTION

A fundamental problem in the design of optical fiber transmission systems is to achieve a desired bit-error rate (BER). Since direct BER measurements can be difficult to obtain, a widely used performance measure is the Q-factor [1], which is used to approximate the BER under the assumption that the electrical currents in the marks and in the spaces at the receiver are Gaussian-distributed. The optical signal-to-noise ratio (OSNR) is another commonly used indicator of performance that is even easier to measure than the Q-factor [2], [3]. However, the relationship between the OSNR and the Q-factor is not straightforward, since the Q-factor depends on the characteristics of the receiver and on the shape of the optical pulses after transmission.

The design and performance evaluation of fiber transmission systems relies just as much on the accuracy and efficiency of receiver models as it does on accurate and efficient transmission modeling [4]. Accurate receiver modeling is especially important when comparing modulation formats. Marcuse [1] and Humblet and Azizoğlu [2] derived widely-used expressions for the Q-factor as a function of either the signal-to-noise ratio (SNR) of the electrical current or the OSNR for a rectangular nonreturn-to-zero (NRZ) optical pulse shape under the assumption that the receiver consists of an optical preamplifier, a rectangular optical filter, a square-law detector, and an integrate-and-dump electrical filter. In this letter, we generalize Marcuse's and Humblet and Azizoğlu's results by deriving a formula for the Q-factor in terms of the OSNR for an arbitrary optical pulse shape immediately prior to the receiver and for arbitrary optical and electrical receiver filters. To do so, we calculate the true moments of the electric current in the receiver, using an approach that was introduced earlier by Winzer *et al.* [4] to calculate the BER. To correctly account for the pulse shape in the formula for the Q-factor we define an enhancement factor, which explicitly quantifies how efficiently the combination of a pulse shape and a receiver translates the OSNR into the SNR of the electrical current in the receiver. We use this method to quantify the advantage in the receiver of using a chirped return-to-zero (CRZ) format rather than a return-to-zero (RZ) or nonreturn-to-zero (NRZ) format.

II. Theory

In this section we derive an expression that relates the Q-factor to the OSNR. We begin by recalling the expressions for the true mean and variance of the electrically filtered current in the receiver as in [4]. We assume that noise from optical amplifiers is the dominant source of noise, as is the case in an optically preamplified receiver [3]. At the input to the receiver, we assume that the signal is polarized, and we let $e_{\rm in}(t)$ denote its scalar-valued electric field, where t is time. We also assume that the noise is unpolarized white Gaussian noise. More specifically, we let $n_{\|_s}(t)$ and $n_{\perp_s}(t)$ be the components of the noise whose polarization states are, respectively, parallel and perpendicular to the polarization state of the signal. These two orthogonal components of the noise are delta-correlated with mean zero and autocorrelation functions given by

$$\langle n_{\|_s}(t)\, n^*_{\|_s}(t')\rangle = \langle n_{\perp_s}(t)\, n^*_{\perp_s}(t')\rangle = N_{\rm ASE}\,\delta\,(t-t')\,, \tag{1}$$

where $N_{\rm ASE}$ is the power spectral density of the noise in each of two orthogonal polarization states. Since the noise is assumed to be unpolarized, the parallel and perpendicular components of the noise are uncorrelated. Our receiver model consists of an optical filter $H_o(\omega)$ with impulse response $h_o(t)$, a square-law photodetector, and an electrical filter $H_e(\omega)$ with impulse response $h_e(t)$. Then the current at the receiver is given by

$$i\,(t) \;=\; R\left\{\left|\left[e_{\rm in}(t)+n_{\|_s}(t)\right]*h_o(t)\right|^2+\left|n_{\perp_s}(t)*h_o(t)\right|^2\right\}*h_e(t), \tag{2}$$

where R is the responsivity of the photodetector and the convolution of two functions $g(t)$ and $h(t)$ is defined by $g(t)*h(t)=\int_{-\infty}^{+\infty}g(\tau)h(t-\tau){\rm d}\tau$.

From (1) and (2) the mean current due to noise is given by $\langle i_n\rangle(t) = \langle i_n\rangle = 2N_{\rm ASE}\, RB_o$, where $\langle\cdot\rangle(t)$ is the average over the statistical realizations of the noise at time t, and $B_o = \int_{-\infty}^{+\infty} |h_o(\tau)|^2 d\tau$ is the power-equivalent spectral width [5] of the optical filter. Here we have assumed that $H_o(0) = H_e(0) = 1$, since the attenuation produced by the filters does not affect the SNR. The variance of the current is given by

$$\sigma_i{}^2(t) = \langle i^2\rangle(t) - \langle i\rangle^2(t) = \sigma^2_{\rm ASE-ASE} + \sigma^2_{\rm S-ASE}(t). \quad (3)$$

The first term on the right side of (3), which is the variance of the current due to noise-noise beating in the receiver, is given by [4]

$$\sigma^2_{\rm ASE-ASE} = 2N^2_{\rm ASE}\, R^2 \int_{-\infty}^{+\infty} |r_o(\tau)|^2\, r_e(\tau){\rm d}\tau, \quad (4)$$

where $r_o(\tau) = \int_{-\infty}^{+\infty} h_o(\tau')h_o^*(\tau+\tau'){\rm d}\tau'$, and $r_e(\tau) = \int_{-\infty}^{+\infty} h_e(\tau')h_e(\tau+\tau'){\rm d}\tau'$ are the autocorrelation functions of the optical and the electrical filters, respectively. The second term, which is the variance of the current due to signal-noise beating in the receiver, is given by

$$\sigma^2_{\rm S-ASE}(t) = 2R^2 N_{\rm ASE} \int_{-\infty}^{+\infty} e_{\rm out}(\tau)h_e(t-\tau) \int_{-\infty}^{+\infty} e^*_{\rm out}(\tau')h_e(t-\tau')r_o(\tau-\tau'){\rm d}\tau'{\rm d}\tau, \quad (5)$$

where $e_{\rm out}(t) = e_{\rm in}(t) * h_o(t)$.

We now use these results to derive a general expression for the Q-factor as a function of the OSNR following a procedure similar to the one described in [1], but using the true mean and the true variance of the electrical current. We start with the standard time-domain definition of the Q-factor, $Q = (\langle i_1\rangle - \langle i_0\rangle)\,/\,(\sigma_1+\sigma_0)$ as in [1]. We define the OSNR by OSNR $= \langle|e_{\rm in}(t)|^2\rangle_t/(2N_{\rm ASE}B_{\rm OSA})$, where $\langle|e_{\rm in}(t)|^2\rangle_t$ is the time-averaged noiseless optical power per channel prior to the optical filter, and $B_{\rm OSA}$ is the power-equivalent spectral width of an optical spectrum analyzer (OSA) that is used to measure the noise power [3].

We find that for any optically preamplified amplitude-shift-keyed system the Q-factor can be expressed as a function of the OSNR as

$$Q = \frac{(1-\alpha_e)\xi\mathrm{OSNR}}{\sqrt{K_1\xi\mathrm{OSNR}+1}+\sqrt{K_0\,\alpha_e\,\xi\mathrm{OSNR}+1}}\sqrt{M}, \tag{6}$$

where ξ is the enhancement factor, K_1 and K_0 are the signal-noise beating parameters of the marks and spaces, respectively, and M is the effective number of noise modes, all of which are unitless. The parameters

$$M = \frac{\langle i_n\rangle^2}{\sigma^2_{\mathrm{ASE-ASE}}} \quad \text{and} \quad K_1 = \frac{\langle i_n\rangle\,\sigma^2_{\mathrm{S-ASE}}\,(t_1)}{i_{1_s}\sigma^2_{\mathrm{ASE-ASE}}} \tag{7}$$

are determined by the shapes of the receiver filters and K_1 also depends on the optical pulse shape immediately prior to the receiver. In the equation for K_1, the time t_1 is the sampling time of a mark, and i_{1_s} is the noiseless current of the marks at time t_1. The definition of K_0 is analogous to that of K_1. The parameter $\alpha_e = i_{0_s}/i_{1_s}$ is the extinction ratio of the electrical current, where i_{0_s} is the noiseless current of the spaces at the sampling time. The parameter α_e can determined from the optical extinction ratio α_o by evaluating (2) without noise. The enhancement factor ξ is the ratio between the signal-to-noise ratio of the electrical current of the marks SNR_1 and the OSNR and is given by

$$\xi = \frac{\mathrm{SNR}_1}{\mathrm{OSNR}} = \frac{i_{1_s}}{\langle i_n\rangle}\frac{2N_{\mathrm{ASE}}B_{\mathrm{OSA}}}{\left\langle|e_{\mathrm{in}}(t)|^2\right\rangle_t} = \xi'\frac{B_{\mathrm{OSA}}}{B_o}, \tag{8}$$

where $\xi' = i_{1_s}/\left(R\langle|e_{\mathrm{in}}(t)|^2\rangle_t\right)$ is the normalized enhancement factor. The enhancement factor quantifies how efficiently the combination of the pulse shape and receiver translates OSNR into SNR of the electrical current of the marks in the receiver. All three parameters, ξ, K, and M, should be taken into account in the optimization of the receiver performance.

To efficiently compute the Q-factor using (6) we use the fast Fourier transform to numerically compute the multiple integrals in (4) and (5) since, by the convolution theorem,

$$\sigma^2_{\mathrm{ASE-ASE}} = N^2_{\mathrm{ASE}} R^2 \int_{-\infty}^{+\infty} \left| \mathcal{F}_\tau^{-1} \left\{ \left| \tilde{H}_o \right|^2 \right\} \right|^2 \mathcal{F}_\tau^{-1} \left\{ |H_e|^2 \right\} \mathrm{d}\tau, \tag{9}$$

where $\tilde{H}_o(\omega) = H_o(-\omega)$, and $\mathcal{F}_\tau[\cdot]$ and $\mathcal{F}_\tau^{-1}\{\cdot\}$ denote the forward and inverse Fourier transform with respect to τ, while

$$\sigma^2_{\mathrm{S-ASE}}(t) = 2R^2 N_{\mathrm{ASE}} \mathcal{F}_t^{-1} \left\{ H_e \, \mathcal{F}_\tau \left[e_{\mathrm{out}}(\tau) \mathcal{F}_\tau^{-1} \left\{ \left| \tilde{H}_o \right|^2 \mathcal{F}_{\tau'} \left[e^*_{\mathrm{out}}(\tau') h_e(t-\tau') \right] \right\} \right] \right\}. \tag{10}$$

III. Numerical Results

We now validate the formula (6) for computing the Q-factor from the OSNR by comparison with Monte Carlo simulations in which the Q-factor is computed using the standard time domain formula. We validate the method using a back-to-back 10 Gbit/s system with optical noise added by a constant spectral density Gaussian noise source. Since our study is focused on the combined effect that the pulse shape and the receiver has on the system performance, we did not include transmission effects. In Fig. 1, we plot the Q-factor versus the OSNR for a CRZ format with an infinite extinction ratio. The electric field of a CRZ pulse is given by $u(t) = U_o \left\{ \left[1 + \cos(2\pi t/T)\right]/2 \right\}^{1/2} \exp\left[iA\pi\cos(2\pi t/T)\right]$, where $A = -0.6$, and T is the bit period [6]. We transmitted the CRZ signal through dispersive fiber with a total dispersion of -126 ps/nm to minimize the width of the pulses prior to the receiver [6]. The receiver consisted of a Gaussian-shaped optical filter with a full-width at half maximum of 124 GHz and a fifth-order electrical Bessel filter with a 3 dB-width of 8.5 GHz. The power-equivalent spectral width of the OSA was 25 GHz.

In Fig. 1, we show the results for our method with a solid line. We obtained these results using only a single mark and a single space of the transmitted bit string. We show the results for the time-domain Monte Carlo method with a dashed line. We obtained these results by averaging over 100 samples of the Q-factor, where in each sample we estimated the means and standard deviations of the marks and spaces using 128 bits. The agreement between the two methods is excellent.

In Fig. 2(a), we use (6) to plot the Q-factor versus the OSNR for the CRZ, RZ and NRZ formats using both an infinite extinction ratio and an optical extinction ratio of 18 dB. With a finite extinction ratio, we use the same pulse shapes in the spaces as in the marks but with a lower power. The RZ pulse shape was determined by setting $A = 0$ in the formula for the CRZ pulse. The rise time of an NRZ pulse was 30 ps. The parameters for the three formats are given in Table I. The normalized enhancement factor is larger for the CRZ format than for the RZ format and is larger for the RZ format than for the NRZ format, due to the decrease in the pulse duration prior to the receiver, as shown in Fig. 2(b). Thus, the CRZ format performs better than the RZ format, which in turn performs better than the NRZ format. The performance is worse with a finite extinction ratio, since optical energy is transfered from marks to spaces, which reduces ξ and increases K_0. In [1]–[3], $K_1 = 2$, since the electrical filter was approximated by an integrate-and-dump filter. For the receiver that we studied, the use of this approximation would overestimate the Q-factor of the CRZ format by 32%.

IV. Conclusions

We have derived an accurate formula that relates the Q-factor to the OSNR for amplitude-shift-keyed optical fiber transmission systems with arbitrary optical pulse shapes and receiver characteristics. We also defined the enhancement factor and two other parameters that explicitly quantify the relative performance of different modulation formats. We validated this method by comparison with Monte Carlo simulations, and we applied it to compare the performance of the CRZ, RZ, and NRZ modulation formats. For the receiver that we studied the CRZ modulation format outperforms both the RZ and the NRZ formats with the same optical power and receiver characteristics because the CRZ format has larger enhancement factor. Since the method is computationally efficient, it can be used for the analysis of transmission systems that have pattern dependences.

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Captions:

Table I: Parameters of the modulation formats used in Fig. 2.

Figure 1: Comparison of the formula (6) with the time domain Monte Carlo method for computing the Q-factor as a function of the OSNR for the CRZ raised cosine format whose pulse shape is shown in Fig. 2(b). The solid line shows the result using (6). The dashed line and the two dotted lines show the mean Q-factor and the confidence interval, defined by the mean Q-factor plus and minus one standard deviation, respectively, computed using the time domain Monte Carlo method.

Figure 2: A performance comparison of the modulation formats whose parameters are given in Table I. (a) The Q-factor as a function of the OSNR. The solid, dashed, and dotted curves show the results with an infinite extinction ratio for the CRZ, RZ, and NRZ formats, respectively. The curves with circles, rectangles and triangles show the corresponding results with an optical extinction ratio of 18 dB. (b) The shapes of an isolated mark for the different formats prior to the receiver. The solid, dashed and dotted curves are results for the CRZ, RZ, and NRZ formats, respectively.

Format	α_o (dB)	ξ'	ξ	K_1	K_0	M
CRZ	$-\infty$	4.20	0.80	3.48	0	21.23
CRZ	-18	4.13	0.78	3.48	2.99	21.23
RZ	$-\infty$	3.21	0.61	3.05	0	21.23
RZ	-18	3.16	0.60	3.05	3.04	21.23
NRZ	$-\infty$	2.12	0.40	2.83	0	21.23
NRZ	-18	2.05	0.39	2.83	2.74	21.23

TABLE I
PARAMETERS OF THE MODULATION FORMATS USED IN FIG. 2.

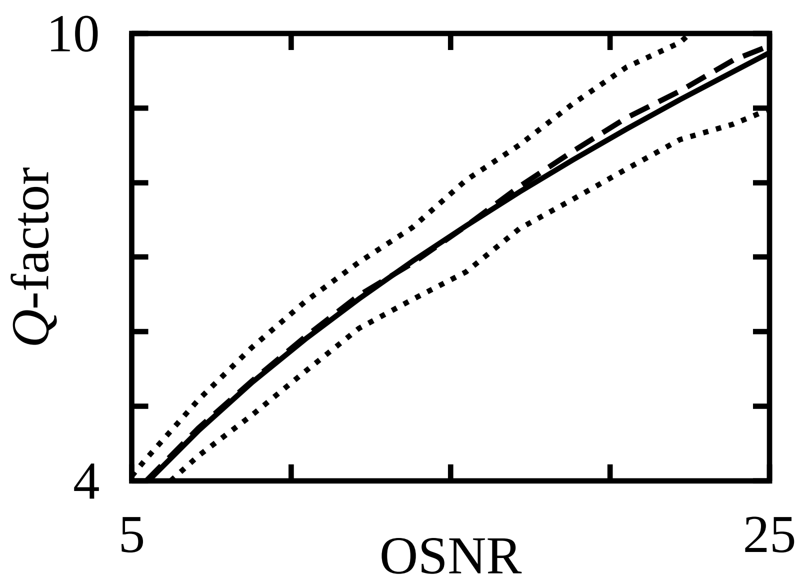

Fig. 1. I. T. Lima, Jr., *et al.*, Performance characterization of chirped return-to-zero modulation format using an accurate receiver model

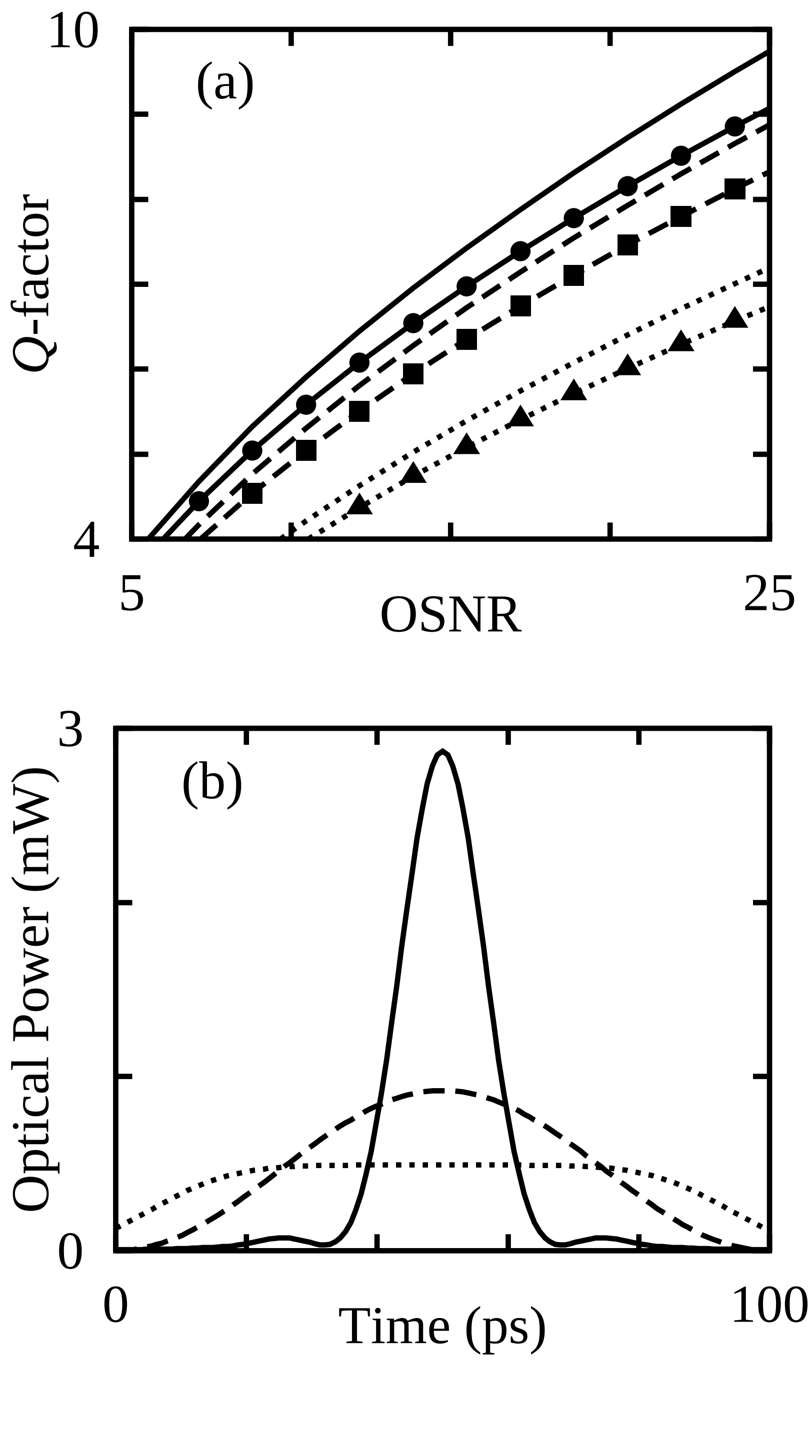

Fig. 2. I. T. Lima, Jr., *et al.*, Performance characterization of chirped return-to-zero modulation format using an accurate receiver model