Project: Numerical errors

GOALS

1. To characterize and understand numerical error in simulations of the nonlinear Schrödinger equation and modifications of it.

REFERENCES

1. O. E. Martinez, R. L. Fork, and J. P. Gordon, "Theory of passively mode-locked lasers for the case of a nonlinear complex-propagation coefficient," J. Opt. Soc. Am. B **2** (5), pp. 753–760, 1985.

2. P. A. Bélanger, "Coupled-cavity mode locking: A nonlinear model," J. Opt. Soc. Am. B **8** (10), pp. 2077–2081, 1991.

3. L. F. Shampine, *Numerical Solution of Ordinary Differential Equations,* CRC Press, 1994.

4. C. William Gear, *Numerical Initial Value Problems in Ordinary Differential Equations*, Prentice Hall, 1971.

PROCEDURE

Consider the modification of the nonlinear Schrödinger equation given by

$$\frac{\partial q}{\partial z} = \frac{i}{2}\frac{\partial^2 q}{\partial t^2} + i|q|^2 q + \gamma q + \frac{c}{2}\frac{\partial^2 q}{\partial t^2}, \tag{1}$$

which describes propagation through a medium with distributed Gaussian-shaped filtering and distributed excess gain. This modification of the nonlinear Schrödinger equation has been used to model pulse propagation in some mode-locked lasers. Equation (1) has an exact solution of the form

$$q(z,t) = A_0 \left[\operatorname{sech}\left(\frac{t-t_0}{\tau}\right)\right]^{1-i\beta} \exp\left(i\psi z\right). \tag{2}$$

1. In the above equation, assume that the filtering strength c is a known quantity, and that the excess gain γ must be set to compensate for the loss due to filtering. In the exact solution, it is possible to specify one of the parameters A_0, τ, β, ψ, or γ, and find the others in terms of it. How do A_0, β, ψ, and γ depend on τ, and how does τ depend on the FWHM pulse width? What are the units of c, and what does it really mean? Show your work.

2. Make a choice for c and τ in the above equation. Modify the OCS code to solve this differential equation with an initial condition given by the above solution. Make plots of the power and of the real part of the electric field versus distance and time for the numerical solution. After, say, 1000 km of transmission, how well does your pulse agree with the exact solution?

3. Use fixed z-steps in your code. Determine the mean-square error in the pulse power as a function of distance for various choices for the z step after 1000 km of transmission. Do the same thing for the mean-square error of the real part of the pulse. Make plots of the error vs. Δz for each, where Δz is 62.5 m, 125 m, 250 m, 500 m, 1 km, 2 km, 4 km, 8 km, 16 km, and 32 km. How do these errors scale with Δz?

4. For a reasonable choice of Δz, how does the error scale with the number of grid points (qtPoints) you use to make up your pulse? Use values of `qtPoints` of 16, 32, 64, 128, 256, 512, 1024, 2048, and 4096. Make similar plots of the dependence of error in the power and real part of q on the number of grid points you choose.

5. Input an unchirped sech pulse (i.e., $\beta = 0$) with the same FWHM pulse width and peak power as you used previously. What happens to this pulse as a function of distance and time for reasonable choices of `qtPoints` and Δz? What is the mean-square difference between this pulse and the sech pulse with the same initial pulse width and peak power as a function of distance? How does the z variation of this difference change if you double the width of your time domain and double the points? Explain what has changed and why it changed.