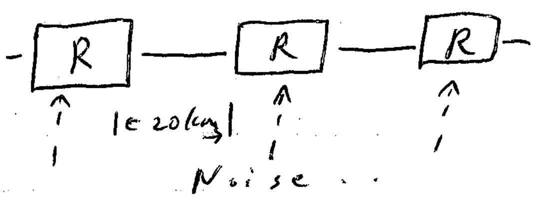
V. Receivers

A. General and Historical Considerations

1\. a. Before the invention of erbium-doped fiber amplifiers (EDFAs) in the late 80's and their deployment in the mid 90's, optical fiber communications systems operated through chains of repeaters space approximately 20 km apart in long-haul systems



The optical fiber introduced no significant impairment except loss. Noise was introduced by the receivers and was due to electrical issues in the receivers, to be discussed later.

b. In this context, it makes sense to characterize receivers by their sensitivity. The sensitivity is the average power required to produce a specified bit error rate (BER) — usually 10^{-9} Sometimes, sensitivity is specified in photons / bit.

$$\bar{P}_R = \frac{\bar{E}_R}{T} = \frac{\bar{N}_R h\nu}{T}$$

where T is the time in one bit slot
$h\nu$ is the energy in one photon

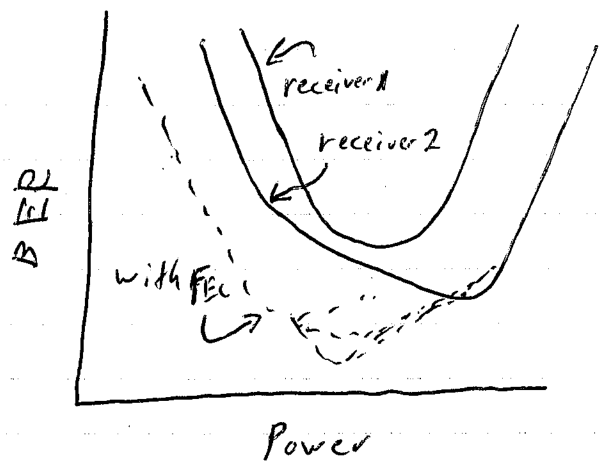
c. This sort of analysis is still useful for (1) short-haul systems, (2) space-based or free-space optical communications. However, it does not make sense for modern-day, long-haul systems.

d. Modern-day, long-haul systems have the following features:

(1) Noise from the EDFAs dominates over noise from the receiver.

(2) Modern receivers typically have an EDFA pre-amplifier, whose noise dominates over electrical noise. Hence, the sensitivity of modern receivers is determined by optical noise

(3) Transmission nonlinearity sets an upper limit to the power. The behavior of BER as a function of power has the following behavior:



The frequency characteristics of the receiver determine the response. We will say more about this later.

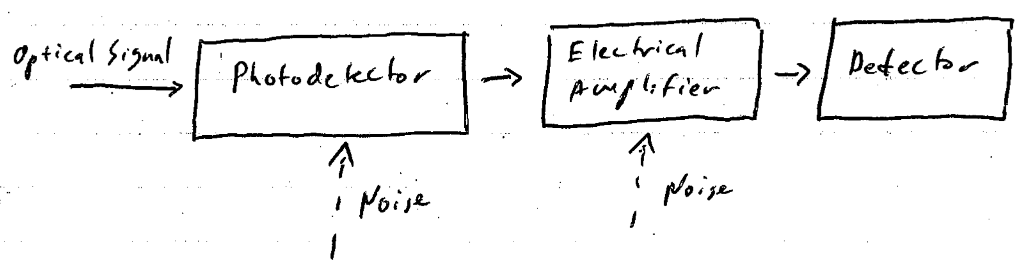
(4) Using different modulation formats (NRZ vs. RZ vs. CRZ) also changes these curves. Different receiver characteristics can work better with different formats
THE TWO MUST BE JOINTLY OPTIMIZED

(5) Signal-processing (e.g. LMS approaches) and forward error correction (FEC) are increasing part of modern receiver design and can effect the curve above (particularly the low end) strongly

e. The bottom line is: (1) frequency analysis is much more important than traditional noise analysis in optimizing receivers. (2) All textbooks of which I am aware are based on the traditional analysis and are not very useful for long-haul optical fiber communications systems. (3) Modern analysis, based on frequency characteristics, only started in the late 90's. While aspects are old, square law detection in the photodiode — more on that later — leads to fundamentally different behavior. [In particular, matched filters are not possible.]

2. Traditional model and characterization (mostly following Kazovsky, et al.)

a. Basic Form



b. Fundamental sensitivity limit: Consider an ideal receiver (no noise) that can count even a single photon. No photons can arrive except in marks [no intersymbol interference (ISI)]. Quantum mechanically, the photon

arrival is a Poisson process.
Hence

$$P[N_R(T)=n] = \frac{(2\bar{N}_R)^n}{n!} e^{-2\bar{N}_R}$$

for $n=0, 1, 2, 3, \ldots$ when a 1 bit is transmitted.

Errors occur when 0 photons are detected when a 1 bit was transmitted

$$p(e) = \frac{1}{2} P[N_R(T)=0] = \frac{1}{2} e^{-2\bar{N}_R}$$

$p(e) = 10^{-9}$ when $\bar{N}_R = 10$ photons/bit
$\simeq -50$ dBm $\subset$ 13 nW at 1.55 μm

c. Some noise sources
(1) thermal noise: (Johnson noise)

$$\sigma_{thermal} = \sqrt{\frac{2kT_o R}{T}}$$

where R is the impedance of the photodiode / amplifier, T_o is the temperature and k is the Boltzmann constant.

$$v_{photon} = \frac{Rq}{T}$$ is the voltage

produced by a single photon

$$\frac{\sigma_{thermal}}{v_{photon}} = \frac{1}{q}\sqrt{\frac{2kT_oT}{R}} \simeq 2{,}500$$

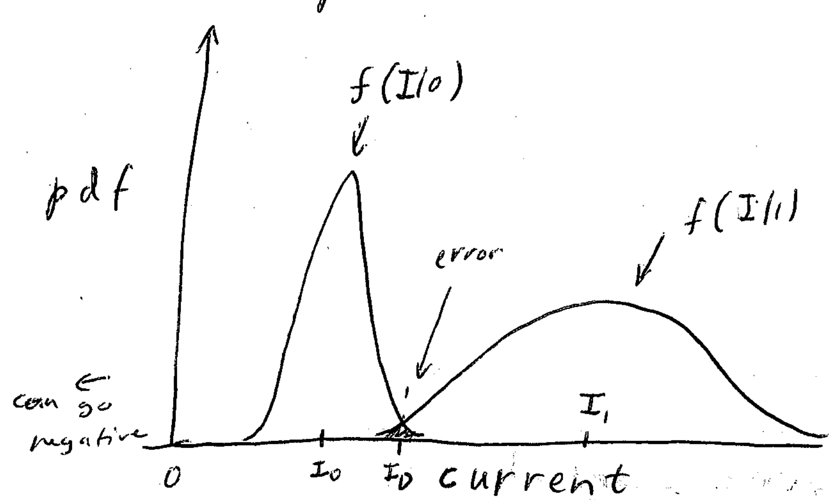
for a 50 ohm impedance and $T_o = 290°K$
which is far above the fundamental sensitivity

(2) dark current (current that flows in the photodiode in the absence of light)

(3) shot noise (current due to the variability in the number of photons received) Also, in an APD, the variability in the number of electrons generated

d. Q-factor and BER
(This concept becomes very important in modern receiver analysis)

It is reasonable, although not always right, to assume that the collection of noise sources in a traditional receiver gives a Gaussian distribution of the prior to the decision: for both marks and spaces



$$BER = \frac{1}{2}\left[P(0|1) + P(1|0)\right]$$

$$P(0|1) = \frac{1}{\sigma_1\sqrt{2}}\int_{-\infty}^{I_D} \exp\left[-\frac{(I-I_1)^2}{2\sigma_1^2}\right] dI$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(\frac{I_1 - I_D}{\sigma_1\sqrt{2}}\right)$$

Similarly,

$$P(1|0) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{I_D - I_0}{\sigma_0\sqrt{2}}\right)$$

$$\Rightarrow \quad BER = \frac{1}{4}\left[\mathrm{erfc}\left(\frac{I_1 - I_D}{\sigma_1\sqrt{2}}\right) + \mathrm{erfc}\left(\frac{I_D - I_0}{\sqrt{2}}\right)\right]$$

The minimum occurs when

$$(I_1 - I_D)/\sigma_1 = (I_D - I_0)/\sigma_0 \equiv \boxed{Q}$$

Explicitly: $\quad I_D = \dfrac{\sigma_0 I_1 + \sigma_1 I_0}{\sigma_0 + \sigma_1}$

and $\quad \boxed{Q = \dfrac{I_1 - I_0}{\sigma_1 + \sigma_0}}$

$$\Rightarrow \quad BER = \frac{1}{2}\,\mathrm{erf}\left(\frac{Q}{\sqrt{2}}\right) \simeq \frac{\exp(-Q^2/2)}{Q\sqrt{2\pi}}$$

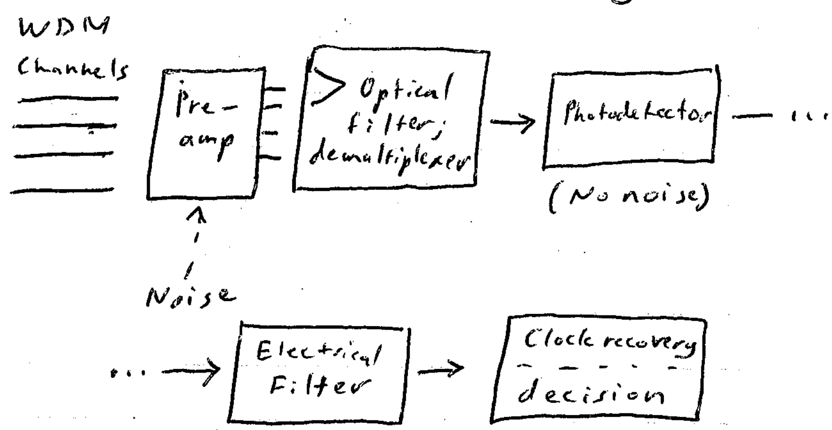
When $BER = 10^{-9}$, $\quad Q = 6$.

|| We still use Q to characterize signals, particularly when BERs are small!

B. Modern Analysis

Note: The design that I will present underlines all aspects of OCS

1. Basic receiver design



Not much can be found on this design chain in textbooks at the present time

2. Pre-amp

$$u_{out}(t) \rightarrow G\, u_{in}(t) + \text{noise}$$

The noise is Gaussian-distributed white noise, calculated as described previously.

If the only source of noise is the pre-amp, then the noise going into the demux is no longer necessarily white because of nonlinear signal-noise interactions during transmission.

3. Optical filter

a. Empirical filters

(1) Passband

$$H_{opt}(f) = \begin{cases} 1, & f_0 - \Delta f < f < f_0 + \Delta f \\ 0, & \text{otherwise} \end{cases}$$

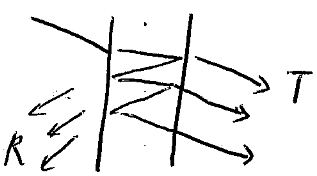
This is the most common numerically, but it is "hard" to realize physically.

(2) Gaussian

$$H_{opt}(f) = \exp\left[-\frac{(f-f_0)^2}{2\sigma_{opt}}\right]$$

b. Physical filters

(1) Fabry-Perot



These are filters made using a partially reflective pair of plate

$$H_{opt}(f) = \frac{(1-A-R)\exp[-2\pi i(f-f_0)\tau]}{1-R\exp[-4\pi i(f-f_0)\tau]}$$

where R = mirror reflectivity
A = internal absorption in one pass one way
τ = one pass in one way delay
f_0 = central frequency

(2) Bragg gratings and AWGs
These can be tailored to produce a variety of shapes

4. Photodector

$$u_{in}(t) \rightarrow I_{out}(t) = \eta\, |u_{in}(t)|^2$$

where $\eta \equiv$ responsivity.
Unless we are trying to make an exact quantitative connection to experiments, we normally just set $\eta = 1$. Otherwise, we have to fit it.

A key issue is that the noise is squared as well:

$$u_{in}(t) = u_{signal}(t) + u_{noise}(t)$$

$$\Rightarrow I(t) = \eta \Big[|u_{signal}(t)|^2 + 2\,\mathrm{Re}\big(u_{signal}(t)\, u^*_{noise}(t)\big) + |u_{noise}(t)|^2 \Big]$$

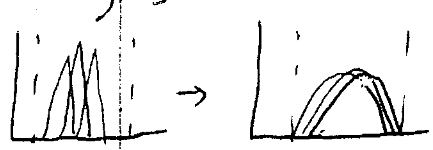
The second term is called spontaneous-signal beat noise and the third term is called spontaneous-spontaneous beat noise. Because of the squaring <u>Gaussian noise becomes non-Gaussian</u>

5.2.4

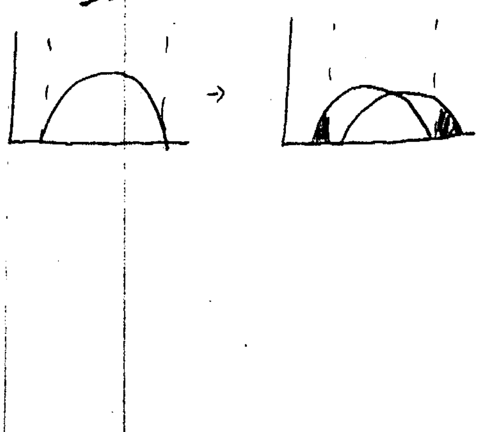
5. Electrical filter

a. Designing the filter right is very important since the optical filter is usually broader than ideal with no way to tailor an ideal shape. Typically the filter has a bandwidth that is 70% - 80% of the data rate, e.g, 7-8 GHz for a 10 Gbs signal. However, in some soliton systems, it was advantageous to use a filter that was only 40% of the data rate.

timing jitter



ISI



(2) A smaller bandwidth eliminates more noise but spreads the pulses more. Spread reduces the effect of timing jitter but can lead to ISI, as shown on the left.

Thus, broader bandwidths are better for broader pulses since broader pulses are less susceptible to timing jitter and more susceptible to ISI.

b. Empirical filter types

(1) Infinite-bandwidth

$$I_{out}(t) = I_{in}(t)$$

which is used to approximate the limit of a broad-bandwidth filter

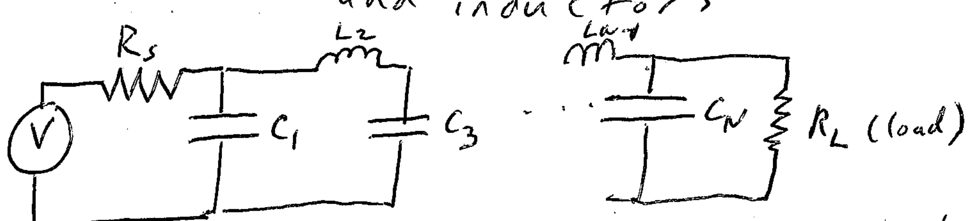
(2) Integrate-and-dump

$$I_{out}(t) = \int_{t_{edge,n}}^{t} I_{in}(t)\, dt$$

where $t_{edge,n}$ is the beginning time of the n-th bit slot

c. Bessel filters

These are very useful analog filters because they have nearly constant time delay and thus lead to low distortion of the signal. They are realizable as a ladder of capacitors and inductors



(odd order - even order has capacitor in the last leg)

Hence, they are widely used in experimental and commercial setups as $N \to \infty$, the time delay becomes constant and the intensity rolloff becomes Gaussian.

In general:

$$H_{elec}(s) = \frac{c_0}{\sum_{n=0}^{N} c_n s^n}$$

where

$$c_n = \frac{(2N-n)!}{2^{N-n}\, n!\,(N-n)!}$$

$$s = -i\alpha_n f / f_{3dB}$$

where f_{3dB} = frequency at which the filter is a factor of 2 lower.

α_n is a numerically determined factor

For a 5^{th} order Bessel filter, which is very common, $\alpha_n = 2.4274$

5.2.7

6. Clock recovery and detection

a. In experiments, there is no tone at the data rate. ← after the filter. and before with NRZ

A typical approach is to obtain a tone at half the data rate, e.g. 5 Ghz for 10 Gbs, using a high-Q resonator or a phase-locked loop. One then frequency doubles. After that, one optimizes the time offset.

b. Computationally, we remove the central group velocity, and we know the group velocities of the other channels very exactly. So, we can recover the clock by either:

(1) taking into account the group velocity shift (zero for central channel)

(2) Using the 10 Ghz tone of the optical signal

(3) Using the 5 Ghz tone and doubling (not implemented)

7. Performance Measures

a. OSNR, Q, and BER are all common measures of performance. BER is the most fundamental, but OSNR and Q are easier to measure. In order to understand how these measures are related and how noise is distributed, we consider the following idealized setting

(1) Input Gaussian white noise
(2) An ideal bandpass optical filter
(3) An ideal signal with an infinite extinction ratio

(4) An integrate-and-dump electrical filter.
(5) Ideal detection at the end of the bit slot

b. We now write the noise during one bit as

$$e(t) = \sum_{n=-\infty}^{\infty} c_n e^{-i\omega_n t} \; ; \quad \omega_n = 2\pi n / T_{bit}$$

The c_n are independent, Gaussian-distributed random variables with zero mean and standard deviation σ that sets the noise level. We next assume that the optical filter passes frequences in the range $\omega_1 < \omega < \omega_1 + 2\pi M/T$ corresponding to a bandwidth $B_{opt} = M/T$

After the optical filter

$$e(t) = \sum_{n=n_1}^{n_1+M} c_n e^{-i\omega_n t}$$

After the photodiode

$$I = \eta \,|E_s(t) + e(t)|^2 \quad \left(\text{where } E_s(t) \text{ is the signal field}\right)$$

After the electrical filter

$$y = \eta \frac{1}{T}\int_0^T I(t)\,dt$$

Writing $E_s(t) = \sum_{n=n_1}^{n+M} s_n e^{-i\omega_n t}$

where the s_n are constant values, we have

$$y = \eta \sum_{n=n_1}^{n_1+M} \left(s_{nr}^2 + s_{ni}^2 + c_{nr}^2 + c_{ni}^2\right)$$

where r and i refer to the real and imaginary parts

We can evaluate the pdfs for y for marks and spaces analytically [see, e.g., Marcuse, JLT, vol. 8, pp. 1816-1823, 1990, Appendix A.

We obtain: For spaces

$$W_0(y) = \left(\frac{1}{2\eta\sigma^2}\right)^M \frac{y^{M-1}}{(M-1)!}\exp\left(-\frac{y}{2\eta\sigma^2}\right)$$

which is exponential, not Gaussian!

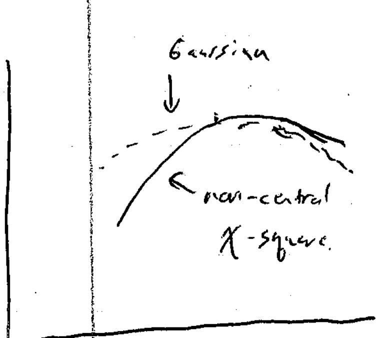
For marks:

$$W_1(y) = \frac{1}{2\eta\sigma^2}\left(\frac{y}{\eta S'^2}\right)^{(M-1)/2} \cdot \exp\left(-\frac{y+\eta S'^2}{2\eta\sigma^2}\right) I_{M-1}\left(\frac{\sqrt{y\eta S'^2}}{\eta\sigma^2}\right)$$

where I_{M-1} is the $(M-1)^{th}$ Bessel function

$S'^2 = \mathcal{E}(s_{ni}^2 + s_{nr}^2)$ = power in the signal (average)

This distribution overshoots a Gaussian at high voltages and undershoots at low voltages and is called non-central χ-square.



This sort of distribution has been observed experimentally.

c. We cannot find the optimal decision point analytically, but it is easy to find numerically.

d. When M is large, both distributions approach Gaussians.

$$W_0(y) \to \frac{1}{\sqrt{2\pi}\,\sigma_0} \exp\left[-\frac{(y-\bar{I}_0)^2}{2\sigma_0^2}\right]$$

$$W_1(y) \to \frac{1}{\sqrt{2\pi}\,\sigma_1} \exp\left[-\frac{(y-\bar{I}_1)^2}{2\sigma_1^2}\right]$$

where $\bar{I}_0 = 2\eta\sigma^2 M$, $\sigma_0 = \bar{I}_0/\sqrt{M}$

$$\bar{I}_1 = \eta(S^2 + 2\sigma^2 M), \quad \sigma_1 = \left(\frac{2\bar{I}_0\bar{I}_1}{M} - \frac{\bar{I}_0^2}{M}\right)^{1/2}$$
$$= I_S + 2\eta\sigma^2 M \qquad = \left(\frac{2\bar{I}_0 I_S}{M} + \frac{\bar{I}_0^2}{M}\right)^{1/2}$$

where $I_S = \eta S^2$ is the signal contribution to $\bar{I}_1$.

$$Q = \frac{\bar{I}_1 - \bar{I}_0}{\sigma_1 + \sigma_0} = \frac{I_S\sqrt{M}}{\bar{I}_0 + (2\bar{I}_0 I_S + \bar{I}_0^2)^{1/2}}$$

Note: $\sigma_1 > \sigma_0$ because of signal-spontaneous beat noise. In commercial systems, this is often much bigger, and

$$Q \simeq \left(\frac{M I_S}{2\bar{I}_0}\right)^{1/2}$$

Ordinarily $M \lesssim 10$. The Gaussian approximation works poorly for the threshold, but it is not too bad for Q in many cases, ~1-2 orders of magnitude off.

S. 2.12

e. Defining $SNR = I_s / I_o$
(which is the electrical signal-to-noise ratio)
we find:

$$Q = \frac{SNR}{\sqrt{2SNR+1}+1}\sqrt{M}$$

In this case, the SNR = OSNR if the bandwidth of the optical filter matches the bandwidth of the spectrum analyzer. Otherwise, we have

$$OSNR = SNR\frac{\Delta f_{OSA}}{\Delta f_{opt\text{-}filter}}$$

8. Considerable work has been done in the last five years to take into account real filter shapes, real pulses, polarization, timing jitter, and non-Gaussian noise. However, there is much to be done.