IV. Transmitters and Amplifiers

A. Transmitters

1. Transmitters come in two basic variants in optical communications systems

   a. $\boxed{\text{Laser diode}}$ A laser diode that is directly modulated. This sort of source is always used in local area networks and almost always used in metropolitan area networks. However, direct modulation usually produces a large and poorly controlled chirp.

   Chirp leads to added bandwidth and, through interaction with dispersion, pulse distortion.

   b. $\boxed{\text{Laser diode}} - \boxed{\text{modulator}}$ A laser diode followed by an external modulator. This configuration eliminates the chirp, but it is more expensive. It is used in core networks. The modulator is $LiNbO_3$ in commercial systems, but other modulators like EAMs (electro-absorption modulators) are used in experimental systems

   c. The extinction ratios in commercial systems are only around 20 dB, which means that the amplitude of the spaces is about 10% of the marks. For some soliton experiments that we do at UMBC, we need a higher extinction ratio, and we use Pritel lasers.

B. Models

1. Most of the pulse models that we use are empirical.

a. Given a pulse shape

$$u_1(z=0,t) \equiv u_1^{(0)}(t)$$

for the marks, we may write

$$u(z=0,t) = \sum_{m=1}^{M} \sum_{n=1}^{N} \alpha_{mn} u_1^{(0)}(t-t_{mn}) \cdot \exp(-i\omega_m t)$$

where ω_m is the frequency of the m-th channel measured with respect to ω_0, M is the total number of channels, N is the total number of bits per channel, α_{mn} is the amplitude of the m-th bit in the n-th channel, and t_{mn} is the central time of the m-th bit in the n-th channel.

b. When we assume an infinite extinction ratio, $\alpha_{mn} = A$ for the marks and equals zero for the spaces, If we take into account a finite extinction ratio we usually just set $\alpha_{mn} = r^{1/2} A$, where r is the extinction ratio. Hence the spaces are just copies of the marks.

c. What I wrote here has to be modified slightly for NRZ to deal with 1-0 and 0-1 transitions, I will explain this modification shortly.

2. Typical empirical models are:

a. Hyperbolic secant (soliton)

$$u_1^{(0)}(t) = \mathrm{sech}\left(\alpha t / t_{FWHM}\right)$$

where $\alpha = 2\ln(\sqrt{2}+1)$

Note $t_{FWHM}/T_{bit} \ll 1$ where T_{bit} is the bit slot duration

b. Gaussian

$$u_1^{(0)}(t) = \exp\left(-\alpha t^2 / t_{FWHM}^2\right)$$

where $\alpha = 2\ln 2$

c. Raised cosine (RZ)

(1) $u_1^{(0)}(t) = \cos\left(\pi t / T_{bit}\right)$

This model goes to zero at the bit boundaries, but it does not take into account the sinusoidal modulation. Better is:

(2) $u_1^{(0)}(t) = \cos\left[\frac{\pi}{2}\sin(\pi t/T_{bit})\right]$

Which has a slightly faster rise at the edges.

d. Chirped return to zero (CRZ)

$$u_1^{(0)}(t) = \cos\left[\frac{\pi}{2}\sin(\pi t/T_{bit})\right] \cdot \exp\left[-i\alpha\pi\cos(\pi t/T_{bit})\right]$$

Note: The literature (and our papers) are a bit confused about the sign of C.

If we define

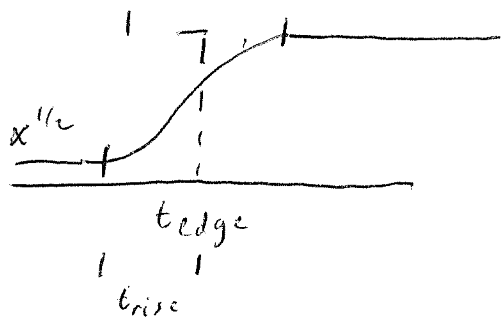
$$C = -\left.\frac{d^2\phi}{dt^2}\right|_{t=0} = \left.\frac{d\omega_{local}}{dt}\right|_{t=0},$$

we find $C = \pi^3\alpha/T_{bit}^2$ is the usual chirp parameter

3. Non-return to zero pulses
   a. Here, bits where a transition occurs must be treated differently from bits where there is no transition since one must round the edge.

4.1.5

One reasonable approach, which corresponds to assuming that the voltage rises linearly in the $LiNbO_3$ modulator is to write the field for a 0-1 transition as



$$u^{(0)} = \alpha^{1/2} \cos\left\{\frac{\pi}{2}\sin\left[\frac{\pi}{4}(t - t_{edge} - t_{rise})/t_{rise}\right]\right\}$$
$$+ 1 - \cos\left\{\frac{\pi}{2}\sin\left[\frac{\pi}{4}(t - t_{edge} - t_{rise})/t_{rise}\right]\right\}$$

b. Other approaches in the literature include:

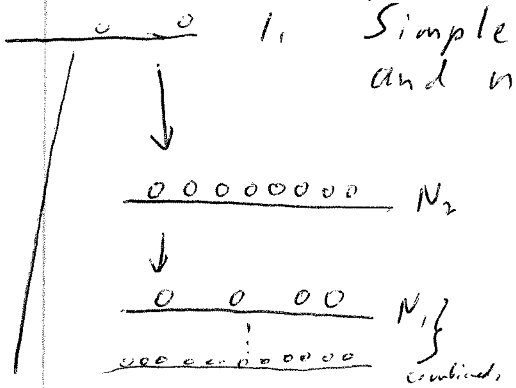
(1) Using a hyperbolic tangent function for the transitions

(2) Using Bessel filters acting on square pulses to round the edges.

c. The 1-0 transition is done symmetrically.

B. Amplifier Models

1. Simple model of amplification and noise



Physically, in order to obtain gain, we must have an inversion between two levels

Gain is proportional to $N_2 - N_1$ which is the rate of stimulated emission minus stimulated absorption

Noise is proportional to N_2 which is the rate of spontaneous emission.

The factor $n_{sp} = N_2/(N_2 - N_1)$ takes into account this ratio [n_{sp} is called the spontaneous emission factor].

b. With constant gain G, one must add $n_{sp}(G-1)$ photons per mode on average into a simulation, This result is a basic result from quantum mechanics.

Each node point corresponds to a mode [frequency wrt ω_0]

The energy per photon is $\hbar(\omega + \omega_0) \simeq \hbar\omega_0$

So, the total energy added to each node point is $\hbar\omega_0 n_{sp}(G-1)$ on average
The corresponding power is:

$$\hbar\omega_0 n_{sp}(G-1)\,\Delta\omega/2\pi$$
$$= \hbar\omega_0 n_{sp}(G-1)/T$$

Consequently: $\Delta|u|^2 = \hbar\omega_0 n_{sp}(G-1)/T = \Delta|\tilde{u}|^2$
on average.

c. In principle, this amount is added in a Gaussian distribution to both the real and imaginary parts of u

$$P_{Re[\Delta u]}\{x\} = \frac{1}{\sqrt{2\pi}\,\sigma}\exp\left[-x^2/2\sigma^2\right]$$
$$\sigma = \left[\frac{1}{2}\hbar\omega_0 n_{sp}(G-1)/T\right]^{1/2}$$

In practice one can use a uniform distribution or even a constant distribution, as long as the expected energy content is the same due to the central limit theorem.

2. Amplifiers with gain saturation (EDFAs). [Gain saturation is very important in stabilizing systems and usually must be taken into account.]

a. Inside the amplifier, we have

$$\frac{\partial g(z,t_p)}{\partial t_p} = \frac{g_0 - g(z,t_p)}{T_{amp}} - \frac{g(z,t_p)\,|u(z,t_p)|^2}{U_{sat}}$$

Here: t_p stands for physical time, not retarded time

T_{amp} is the relaxation time of the amplifier

We can immediately integrate this equation to yield

$$g(z,t_p) = g_0 \exp\left[-\frac{t_p}{T_{amp}} - \frac{1}{U_{sat}}\int_0^{t_p} |u(z,t_p')|^2 dt_p'\right]$$
$$+ \frac{g_0}{T_{amp}}\int_0^{t_p} dt_p' \exp\left[-\frac{(t_p-t_p')}{T_{amp}} - \frac{1}{U_{sat}}\int_{t_p'}^{t_p} |u(z,t_p'')|^2 dt_p''\right]$$

In practice, $T_{amp} \sim 10$ ms for EDFAs

while we may keep on the order of 100 bits of 100 ps each in a simulation, equalling 10 ns

Hence, we average over the bits in a time domain simulation, replacing

$\int_0^{t_p} |u(z,t_p')|^2 dt_p'$ with $\bar{P}(z)\, t_p$

We thus obtain

$$g(z,t_p) = g_0 \exp\left[-\left(\frac{1}{T_{amp}} + \frac{\bar{P}(z)}{U_{sat}}\right)t_p\right]$$
$$+ \frac{g_0}{1 + \frac{T_{amp}\bar{P}(z)}{U_{sat}}}\left\{1 - \exp\left[-\left(\frac{1}{T_{amp}} + \frac{\bar{P}(z)}{U_{sat}}\right)t_p\right]\right\}$$

4.2.4

Write
$\bar{P}(L) = G\,\bar{P}(0)$

Then, integrating

$$G = G_0 \exp\left[-\frac{T_{amp}\,\bar{P}(0)}{U_{sat}(G-1)}\right]$$

See, Kazovsky, et al., p. 413

The steady-state solution is

$$g(z) = \frac{g_0}{1 + \frac{T_{amp}\,\bar{P}(z)}{U_{sat}}}$$

b. From the steady state solution, we may integrate

$$\frac{d\,\bar{P}(z)}{dz} = g(z)\,\bar{P}(z)$$

which can be integrated numerically to obtain the final gain. (It also has transcendental expression)

c. There are situations where transience is important, and we must take into account the slow variations (on the order of a millisecond) in $P(z)$

(1) In recirculating loops, the same EDFAs are used continuously and thus interact with the same train over milliseconds

(2) In networks, when channels are added and dropped, fluctuations result.

3. Noise with gain saturation.

a. The gain varies because the populations in the levels vary. As a consequence n_{sp} is not constant.

Writing $G = \int_0^L g(z')\,dz'$,

where L is the length of the amplifier, we can use an average $n_{sp} = \overline{n}_{sp}$ in our previous formula.

Often, n_{sp} is not know well a priori. What is known is the OSNR at the end of the amplifier and the gain. In that case, this is good enough.

b. A more careful calculation takes into account the change in the population densities

Noting that $N_2 = \frac{N_0}{2} + \frac{N_2 - N_1}{2}$

where $N_0 = \frac{N_1 + N_2}{2}$ determines the small signal gain g_0

we conclude that

$$\Delta U_{noise} \propto \left(\frac{g_0}{2} + \frac{g(z)}{2} \right)$$

$$\left[\text{Or } n_{sp} = \frac{g_0 + g(z)}{2g(z)} \right]$$

Hence, $$\frac{dS}{dz} = \hbar\omega_0 \left[\frac{g_0}{2} + \frac{g(z)}{2} \right]$$

gives the expected change in energy/mode along the amplifier.

4. So far, we have neglected the variation of the gain with frequency. In EDFAs, what we call the levels 1 and 2 are actually a complicated mix of levels. A major simplification is that they can be considered homogeneous. Energy put into the lines ~~at~~ any frequency does not effect the absorption or emission profile of the line — just the total population in the line.

In this case, one must seperately account for the gain due to absorption and the loss due to emission

4.2.7

Writing $\alpha_a(\omega) = \sigma_a(\omega)\, \Gamma(\omega)\, N_0$

σ_a = absorption cross section

σ_e = emission cross section

Γ = overlap integral

$\alpha_e(\omega) = \sigma_e(\omega)\, \Gamma(\omega)\, N_0$

for the gain and loss, we have:

$$G(\omega) = \exp\left[\int_0^L \left(\alpha_a(\omega) \frac{N_2(z)}{N_0} - \alpha_e(\omega) \frac{N_1(z)}{N_0} \right) dz \right.$$

We can determine N_1 and N_2 by appropriate integration over the power and fiber parameters.

[See Kazovsky, et al., pp. 441-442