Polarization decorrelation in optical fibers with randomly varying birefringence

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Polarization decorrelation in single-mode fibers with randomly varying birefringence is studied. We find that decorrelation length is minimized for a given beat length if the average autocorrelation length of the birefringence is close to the average beat length. The differential time delay between the polarization modes is found to depend on the autocorrelation length of the birefringence rather than on the decorrelation length of the polarization modes.

Even the best so-called single-mode communication fibers are birefringent. The orientation and the strength of the birefringence shift randomly, scattering light from one local polarization eigenstate to another. The polarization dispersion that results will limit the transmission rate in both linear[1] and soliton systems.[2] The key physical parameters that determine the rate at which light pulses spread in linear systems[1] or the rate at which solitons lose energy in the nonlinear systems[2] are the polarization decorrelation length h_E, the average beat length L_B, and the autocorrelation length of the birefringence fluctuations in the fiber, h_{fiber}. The parameter h_E, which also depends on both L_B and h_{fiber}, is the length scale over which the electric field loses memory of its initial distribution between the local polarization eigenstates and can be treated as random. It is possible to derive analytical formulas that relate h_E to L_B and h_{fiber} in some special cases, including the diffusion limit studied by Ueda and Kath,[3] in which $h_{\text{fiber}} \ll L_B$, and the weak-coupling limit studied by Poole,[1] in which $L_B \ll h_{\text{fiber}}$.[4] Neither of these limits necessarily holds in communication fibers. We will therefore use simulations to determine $h_E(h_{\text{fiber}}, L_B)$, which is the purpose of this Letter.

To carry out this investigation, we must settle on definite models for the underlying birefringence fluctuations. Here we confront the difficulty that little is experimentally known about these fluctuations. We have therefore used two physically reasonably models and investigated their consequences. In the first model we permit the birefringence orientation to vary randomly but keep the strength fixed; in the second model we permit both to vary. In both cases we find that the qualitative behavior is similar and that h_E is minimized when $h_{\text{fiber}} \simeq L_B$. Explicitly the spatial dependence of the electric field $\mathbf{E}(r, \omega, z)$ is given by

$$\frac{\partial \mathbf{E}(r, \omega, z)}{\partial z} = i\begin{pmatrix} \beta_1 + i\alpha/2 & y \\ y^* & \beta_2 + i\alpha/2 \end{pmatrix}\mathbf{E}(r, \omega, z), \tag{1}$$

where β_1 and β_2 are real constants, α is fiber loss, and z is the distance along the fiber. Substitution of $\mathbf{E}(r, \omega, z) = \exp(-\alpha z/2 + i(\beta_1 + \beta_2)/2)\mathbf{U}(r, \omega, z)$ into Eq. (1) yields

$$\frac{\partial \mathbf{U}(r, \omega, z)}{\partial z} = i\begin{pmatrix} x & y \\ y & -x \end{pmatrix}\mathbf{U}(r, \omega, z), \tag{2}$$

where $x = (\beta_1 - \beta_2)/2$. The beat length is given by $L_B = 2\pi/(\beta_1 - \beta_2)$. The coupling constant y is taken to be real because the fiber is assumed to be linearly birefringent. In our calculations both x and y are random variables. In the first model we assume that

$$x = b\cos\theta, \qquad y = b\sin\theta, \tag{3}$$

where b is constant. Hence the strength of the birefringence remains the same, while the orientation of the birefringence axes varies randomly. Instead of a sudden twisting of the orientation axes, we assume that the rate of change of the angle θ is a white-noise process,[5] i.e.,

$$\frac{d\theta}{dz} = g(z), \qquad \langle g(z)\rangle = 0, \qquad \langle g(z)g(z+u)\rangle = \Gamma\delta(u), \tag{4}$$

where Γ is a constant and $\delta(u)$ is the Dirac delta function. It follows from the central-limit theorem that the distribution of $g(z)$ does not matter much. When $g(z)$ is Gaussian distributed, then $\theta(z)$ is a Wiener process, and the distribution function of θ is given by

$$f(\theta) = \frac{1}{[2\pi\Gamma(z - z_0)]^{1/2}} \sum_{n=-\infty}^{+\infty} \exp\left[-\frac{(\theta + 2n\pi)^2}{2\Gamma(z - z_0)}\right], \tag{5}$$

where we assume $\theta = 0$ at $z = z_0$. Given any distribution of $g(z)$, $f(\theta)$ is well approximated by Eq. (5) at long distances. The ensemble average of the coefficients x and y is given by

$$\langle x(z)\rangle = b\langle\cos\theta(z)\rangle = b\exp[-\Gamma(z - z_0)/2], \tag{6}$$

$$\langle y(z)\rangle = b\langle\sin\theta(z)\rangle = 0. \tag{7}$$

Hence the random variation of the coefficients falls off with a characteristic length given by $2/\Gamma$. The strength of the birefringence is $\langle x^2 + y^2\rangle = b^2$, as

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expected. In the second model both x and y vary independently according to the following Langevin equations:

$$\frac{dx}{dz} = -\alpha x + g(z),$$
$$\frac{dy}{dz} = -\alpha y + h(z), \tag{8}$$

where α is a constant and both $g(z)$ and $h(z)$ are white-noise processes with zero mean and the same distribution. The first two moments of the coefficients x are

$$\langle x(z)\rangle = 0, \tag{9}$$
$$\langle x^2(z)\rangle = (\langle {x_0}^2\rangle - \Gamma/2\alpha)\exp[-2\alpha(z - z_0)] + \Gamma/2\alpha, \tag{10}$$

where we assume $\langle x(z_0)\rangle = 0$. If we choose $\langle {x_0}^2\rangle = \Gamma/2\alpha$, then $x(z)$ is a stationary process. The autocorrelation function of $x(z)$ is then given by

$$\langle x(z + u)x(z)\rangle = \langle {x_0}^2\rangle \exp(-\alpha u). \tag{11}$$

Hence $x(z)$ and similarly $y(z)$ have a finite autocorrelation length given by $1/\alpha$. The average birefringence is $\langle x^2 + y^2\rangle = \Gamma/\alpha$.

Equation (2) is solved with Eqs. (3) and (4) or Eq. (8). We measure the polarization decorrelation length h_E, using the ensemble average of the Stokes parameter $\langle s_1(z)\rangle = \langle |\mathbf{U} \cdot \hat{e}_1(z)|^2 - |\mathbf{U} \cdot \hat{e}_2(z)|^2\rangle$, where $|\mathbf{U}| = 1$. There are two physically sensible choices of the orthogonal unit vectors $\hat{e}_1(z)$ and $\hat{e}_2(z)$. We may choose them to equal $\hat{e}_1(z_0)$ and $\hat{e}_2(z_0)$, the polarization eigenstates at the beginning of the simulations, or we may choose them to equal the local polarization eigenstates. We start our simulations with $\mathbf{U}(z_0) = \hat{e}_1(z_0)$, so that $\langle s_1(z_0)\rangle = 1$, and we define h_E so that it equals the length at which $\langle s_1(z_0)\rangle$ falls to $1/e$ of its initial value. When $h_{\text{fiber}}/L_B \ll 1$, we find that the electric field tends to average over the birefringence fluctuations, and use of the initial polarization eigenstates for $\hat{e}_1(z)$ and $\hat{e}_2(z)$ yields a larger value for h_E. When $h_{\text{fiber}}/L_B \gg 1$, we find that the electric field tends to follow the local axes, at least to some extent, and use of the local polarization eigenstates yields a larger value for h_E. It is the larger value that is physically meaningful because h_E corresponds to loss of memory of the initial state, so that the Stokes parameter $\langle s_1(z)\rangle$ should tend to zero, regardless of what eigenstates are used. We carry out our ensemble average by repeating our integration of Eq. (2) 1000 times, using different randomly generated inputs for $g(z)$ in our first model or for $g(z)$ and $h(z)$ in our second model and then averaging our results. We typically use uniform distributions for these functions for computational convenience, but we have verified that the use of a Gaussian distribution leads to no detectable difference. The physical requirements for this ensemble average to correspond to an average over a length of optical fiber are discussed in Ref. 4.

In Fig. 1 we plot the variation of $\langle s_1(z)\rangle$ versus distance for $h_{\text{fiber}} = L_B$ in the first model. We find that, when measured with respect to the local polarization eigenaxes, $\langle s_1(z)\rangle$ decays exponentially with a length scale that is the same as that of the random variations. We can show analytically that this result holds for all values of h_{fiber}/L_B. When measured with respect to the initial eigenaxes, $\langle s_1(z)\rangle$ oscillates as it decays to zero. The decorrelation length $h_{E,\text{fixed}}$ is found to be $0.54L_B$. In Fig. 2 we plot the decorrelation length of $\langle s_1(z)\rangle$ for different values of h_{fiber}/L_B in the first model. The open circles represent measurements with respect to the local eigenaxes, whereas the crosses represent measurements with respect to the initial axes. The dotted curve shows the results from the diffusion limit.[3] When measured with respect to the local eigenaxes, the decorrelation length $h_{E,\text{local}} = h_{\text{fiber}}$. We find that, when measured with respect to the initial eigenaxes, $h_{E,\text{fixed}}$ approaches the values given by the diffusion limit when $h_{\text{fiber}} \ll L_B$ and approaches $0.5h_{\text{fiber}}$ when $h_{\text{fiber}} \gg L_B$. We also find that $h_E = \max(h_{E,\text{local}}, h_{E,\text{fixed}})$ has its

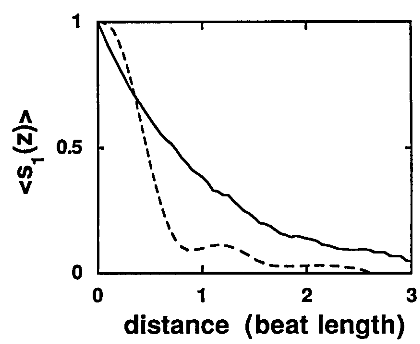

Fig. 1. Variation of the Stokes parameter $\langle s_1(z)\rangle$ versus distance for $h_{\text{fiber}} = L_B$ in the first model when measured with respect to the local polarization eigenaxes (solid curve) and with respect to the initial eigenaxes (dashed curve).

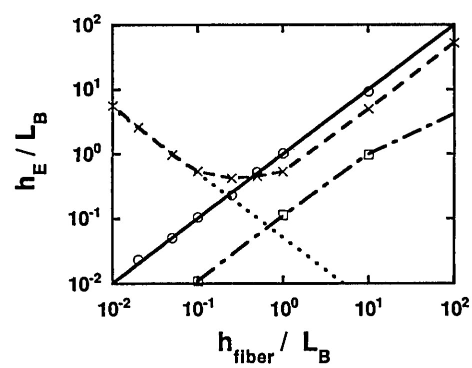

Fig. 2. Decorrelation length of $\langle s_1(z)\rangle$ and the polarization-mode dispersion versus h_{fiber}/L_B for the first model. The open circles represent the decorrelation length measured with respect to the local polarization eigenaxes, and the crosses represent measurement with respect to the initial axes. The dotted curve is the decorrelation length from the diffusion limit. The dotted–dashed curve is the relative differential time delay between the polarization modes from theoretical calculations, and the open squares are the results from numerical simulations.

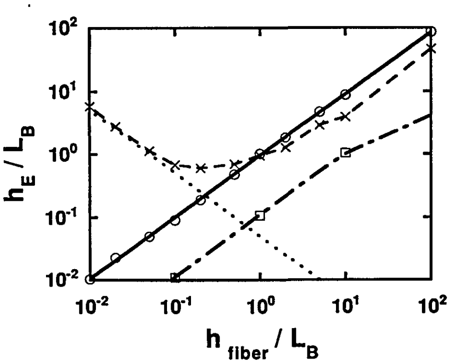

Fig. 3. Decorrelation length of $\langle s_1(z)\rangle$ and the polarization-mode dispersion versus h_{fiber}/L_B for the second model. The notation is the same as that used in Fig. 2.

minimum when $h_{\text{fiber}} \simeq L_B$. In Fig. 3 we plot the decorrelation length of $\langle s_1(z)\rangle$ versus h_{fiber}/L_B for the second model. The results are similar to those of the first model shown in Fig. 2; thus the inclusion of the variation of the birefringence strength has little effect on the decorrelation length h_E.

Finally, we calculate the effect of polarization dispersion in both models by calculating $\langle \tau^2\rangle$, the differential time delay between the polarization modes. For both models it can be shown by use of the approach discussed by Foschini and Poole[6] that

$$\langle \tau^2\rangle = 8\langle k'^2\rangle {h_{\text{fiber}}}^2[z/h_{\text{fiber}} + \exp(-z/h_{\text{fiber}}) - 1], \quad (12)$$

where $k' = \partial k/\partial \omega$. We shall present the complete calculation elsewhere. In the first model, $\langle k'^2\rangle = k'^2$ because the magnitude of the birefringence does not depend on distance. The differential time of flight is independent of the frame of its measurement. From Eq. (12), $\langle \tau^2\rangle$ depends only on h_{fiber} and L_B but not h_E. We note that $h_{E,\text{local}} = h_{\text{fiber}}$ in both models. We assume that the pulse is nearly monochromatic, so that the propagation constant varies linearly with frequency, i.e., $\langle k'^2\rangle \propto \langle k^2\rangle$. At large distances, $\langle \tau^2\rangle \approx h_{\text{fiber}}z/{L_B}^2$. In Figs. 2 and 3 we plot the relative differential time delay between the polarization modes for the first and the second models, respectively, after $100L_B$. The dotted–dashed curves represent the results from Eq. (12), and the open squares represent results from numerical simulations. We find that the simulated results agree well with the calculated results.

In conclusion, using two physically reasonable models, we show that the decorrelation length h_E of the electric fields has its minimum when $h_{\text{fiber}} \simeq L_B$. However, the polarization-mode dispersion is found to depend only on the ratio of the autocorrelation length of the birefringence and the beat length; it does not depend on the decorrelation length of the electric fields.

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