Soliton propagation with up- and down-sliding-frequency guiding filters

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We analyze the effect of the third-order guiding filter term on soliton transmission in optical fibers. We find that this term causes a significant difference between the regimes of up- and down-sliding of filter frequency. In particular, the use of up-sliding requires less additional amplifier gain than down-sliding does, which is preferable in real systems.

Soliton transmission with sliding-frequency guiding filters significantly improves the quality of transmission over transoceanic distances.[1–3] In a transmission line that uses Fabry–Perot étalon filters with a mirror spacing d and a reflectivity R, the distributed filter function is

$$F(\omega) = \frac{1}{l_f}\ln\left\{\frac{1-R}{1-R\exp[i(\omega-\omega_f)2d/c]}\right\}, \qquad (1)$$

where l_f is the filter separation. To date, soliton propagation with filters has usually been studied theoretically by expansion of Eq. (1) in a Taylor series as a function of ω and truncation of the series at the quadratic term.[1,4] This theoretical approach reveals no difference between up-sliding, for which $\omega_f' > 0$, and down-sliding, for which $\omega_f' < 0$, whereas in fact experiments to date indicate that up-sliding is clearly preferable.[5] In this Letter we show that the third-order contribution in the Taylor expansion of Eq. (1) leads to an additional offset in the soliton mean frequency. As a consequence, the use of up-sliding requires less amplifier gain at a fixed rate of sliding than does down-sliding and is preferable.

Keeping terms through the third order in the Taylor expansion of Eq. (1), we obtain the propagation equation in soliton units,

$$\frac{\partial u}{\partial z} = i\left(\frac{1}{2}\frac{\partial^2 u}{\partial t^2} + |u|^2 u\right) + \frac{1}{2}\alpha u - \eta_2\left(i\frac{\partial}{\partial t} - \omega_f\right)^2 u - i\eta_3\left(i\frac{\partial}{\partial t} - \omega_f\right)^3 u, \qquad (2)$$

where

$$\eta_2 = \frac{1}{2}\frac{R}{(1-R)^2}\frac{8\pi}{Dl_f c}\left(\frac{d}{\lambda}\right)^2,$$
$$\eta_3 = \frac{2d}{c}\frac{(1+R)}{(1-R)}\frac{1}{3t_0}\eta_2,$$
$$\omega_f' = \frac{d\omega_f}{dz} = \frac{4\pi^2 f' c t_0^{\,3}}{\lambda^2 D},$$
$$\alpha = \frac{\alpha_R t_0^{\,2} 2\pi c}{\lambda^2 D}. \qquad (3)$$

The parameters f' and α_R are, respectively, the sliding rate and the excess gain, D is the fiber dispersion, $t_0 = \tau/1.763$, where τ is the soliton pulse FWHM, and λ is the wavelength. We consider only the fixed sliding rates $\omega_f' = \text{const}$. Note that the third-order term is inversely proportional to pulse duration, so that it becomes increasingly important as the bit rate increases. For typical parameters in current experiments, $d = 1.5$ mm, $R = 9\%$, and $\tau(\text{FWHM}) = 14$ ps, we find that $\eta_3 = 0.5\eta_2$, so that its contribution is not negligible.

We begin by introducing the following *ansatz* for the soliton pulse shape into Eq. (2):

$$u = A\,\text{sech}(At - qz) \times \exp\left[i\frac{1}{2}A^2 z + 3i\eta_3 A\tanh(At - qz) - i\Omega t\right], \qquad (4)$$

where $q = -A(\Omega + \eta_3 A^2)$. Equation (4) is the soliton solution of Eq. (2) to the first order in η_3 and Ω, when $\alpha = \eta_2 = \omega_f' = 0$. We have not included the first-order contributions of α and η_2 in Eq. (4) because they significantly complicate Eq. (4) while having no effect on the frequency offset. The mean frequency ω_0 of the soliton given by Eq. (4) is related to Ω by the relation

$$\omega_0 = \frac{\text{Im}\int dt u\partial u^*/\partial t}{\int dt|u|^2} = \Omega - 2\eta_3 A^2. \qquad (5)$$

The term $2\eta_3 A^2$ describes the frequency shift that is due to the change in shape of the soliton.[6] The total shift of the soliton mean frequency ω_0 is determined by both the second- and third-order filter terms.

Substituting Eq. (4) into Eq. (2), we obtain through the first-order perturbation theory[7] the following pair of coupled equations for the amplitude A and mean frequency ω_0 of the soliton:

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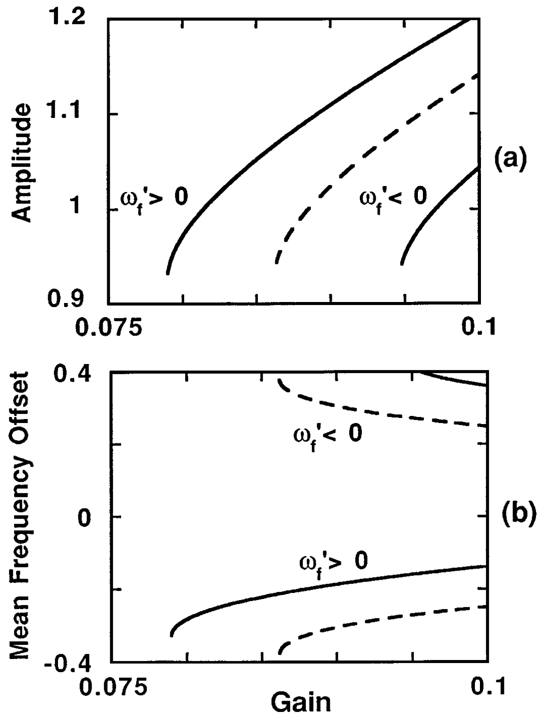

Fig. 1. Equilibrium values of (a) soliton amplitude A and (b) mean frequency offset from the filter frequency $\Delta\omega$ versus excess gain α, as determined from Eqs. (6) and (7). The filter parameters are $\eta_2 = 0.1$, $\eta_3 = 0.05$, and $|\omega_f'| = 0.44\eta_2$. The dashed curves represent the equilibrium values (A^2, ω) for $\eta_3 = 0$.

$$\frac{\mathrm{d}\omega_0}{\mathrm{d}z} = -\frac{4}{3}\eta_2 A^2\left[(\omega_0 - \omega_f) - \frac{6}{5}\eta_3 A^2\right], \quad (6a)$$

$$\frac{\mathrm{d}A}{\mathrm{d}z} = \alpha A - 2\eta_2 A\left[(\omega_0 - \omega_f)^2 + \frac{1}{3}A^2\right]. \quad (6b)$$

At equilibrium, assuming a constant rate of sliding, we find after setting $\mathrm{d}\omega_0/\mathrm{d}z = \omega_f'$ in Eq. (6a) that

$$(\omega_0 - \omega_f) = \frac{6}{5}\eta_3 A^2 - \frac{3}{4\eta_2 A^2}\omega_f'. \quad (7)$$

When $\omega_f' > 0$, corresponding to up-sliding, then the two terms on right-hand side of Eq. (7) are of the opposite sign and the third-order term diminishes the frequency offset that is due to sliding. Physically, sliding leads to an offset because the soliton has inertia so that its central frequency lags behind the central frequency of the filter as it slides. At the same time, the filter is asymmetric because of the third-order term so that the central frequency of the soliton in the absence of sliding is offset from the central frequency of the filter, which is also the frequency corresponding to minimum loss for the soliton. In the presence of up-sliding, the effect of the third-order term is to bring the soliton closer to the minimum loss point and diminish the loss that it experiences. By contrast, when $\omega_f' < 0$, corresponding to down-sliding, the effect of the third-order term adds to the effect of sliding and pushes the soliton farther away from the minimum loss point. This difference is of practical significance because the extra loss in the case of down-sliding implies that extra amplifier gain is needed, which in turn implies that amplifier-induced noise will be larger than in the case of up-sliding, assuming that the sliding rate is the same for both up- and down-sliding.[1] Thus, larger sliding rates can be tolerated in the case of up-sliding than in the case of down-sliding.

Figure 1 show the dependence of the equilibrium values of A and $\Delta\omega = \omega_0 - \omega_f$ on the gain α. When α is below some critical value α_{cr}, there is no stable soliton solution.[1] Mathematically this is revealed by the appearance of an imaginary part in the equilibrium values of A and $\Delta\omega$ below some critical value α_{cr} of the excess gain α. As our approach assumes that A and $\Delta\omega$ are real, we conclude that there is no equilibrium when an imaginary part is present in the solution for A and $\Delta\omega$. In Fig. 1 the curves stop at the points where the equilibrium disappears. When the η_3 term is not taken into account, α_{cr} is determined only by the absolute value of the sliding rate, and there is no difference between up- and down-sliding regimes. The third-order term causes an asymmetry between up- and down-sliding. With up-sliding, stable soliton propagation can be achieved when α_{cr} diminishes, whereas with down-sliding α_{cr} increases.

To confirm the analytical predictions of the first-order perturbation theory, we solved Eq. (2) numerically. As an initial condition we set $u = \mathrm{sech}(t)$, and we then studied its evolution to its equilibrium state. The filter parameters were $\eta_2 = 0.1$ and $\eta_3 = 0.05$, and the sliding rate was set at $|\omega_f'| = 0.44\eta_2$, which is close to the critical value.[1] For both up- and down-

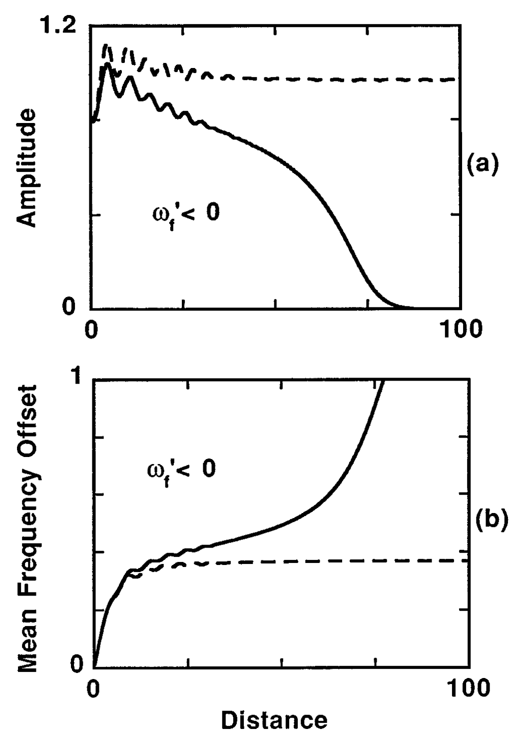

Fig. 2. (a) Soliton amplitude A (b) mean frequency offset from the filter frequency $\Delta\omega$ as a function of distance z given in soliton periods, as determined by numerical solution of Eq. (2) with $\omega_f' = -0.44\eta_2$, corresponding to down-sliding. The solid curves correspond to the gain value α_1 given by Eq. (8), and the dashed curves correspond to α_2 given by Eq. (9).

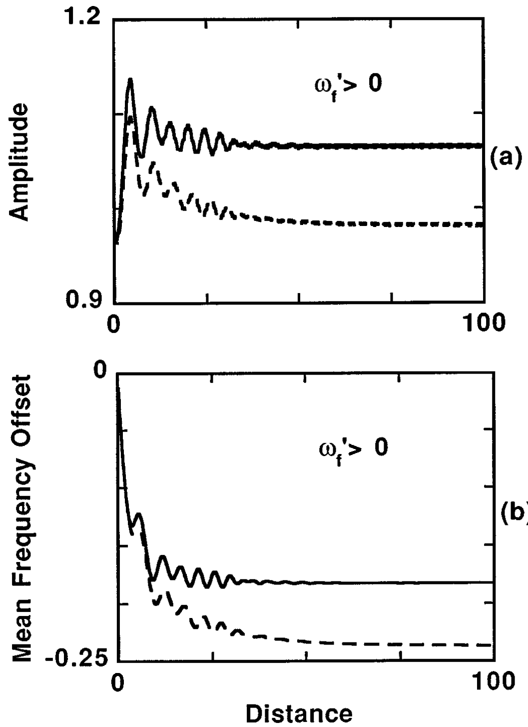

Fig. 3. (a) Soliton amplitude A and (b) mean frequency offset from the filter frequency $\Delta\omega$ as a function of distance z given in soliton periods, as determined by numerical solution of Eq. (2) with $\omega_f' = 0.44\eta_2$, corresponding to up-sliding. The solid curves correspond to the gain value α_1 given by Eq. (8), and the dashed curves correspond to α_2 given by Eq. (9).

sliding we performed two sets of computation, the first set with a gain value $\alpha = \alpha_1$, where

$$\alpha_1 = \frac{2}{3}\eta_2 + 2\eta_2\left(\frac{3}{4\eta_2}\omega_f'\right)^2, \qquad (8)$$

which would lead to equilibrium at $A = 1$ if the third-order contribution were neglected, and the second set with $\alpha = \alpha_2$, where

$$\alpha_2 = \frac{2}{3}\eta_2 + 2\eta_2\left(-\frac{3}{4\eta_2}\omega_f' + \frac{6}{5}\eta_3\right)^2, \qquad (9)$$

which would lead to equilibrium with $A = 1$ when the third-order contribution is included. In all cases we keep the third-order contribution in the simulations. With down-sliding, there is no stable soliton propagation when $\alpha = \alpha_1$; the gain value is below α_{cr} for this regime of propagation and the pulse disappears after some distance of propagation, as shown in Fig. 2. When $\alpha = \alpha_2$, which is greater than α_1, the soliton propagates stably at values of A and ω_0 that are in agreement with the predictions of Eq. (6) and (7). The results for up-sliding are shown in Fig. 3. Stable soliton propagation is observed both when $\alpha = \alpha_1$ and $\alpha = \alpha_2$.

We have examined the influence of the third-order filter contribution on solitons in a system with sliding-frequency guiding filters. We have shown that the soliton loss is lower in a system with up-sliding than it is in a system with down-sliding at the same sliding rate. Consequently, it is advantageous to use up-sliding. This result is consistent with current experimental practice.

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