Accurate Probabilistic Treatment of Bit-Pattern-Dependent Nonlinear Distortions in BER Calculations for WDM RZ Systems

Oleg V. Sinkin, *Member, IEEE*, Vladimir S. Grigoryan, *Member, OSA*, and Curtis R. Menyuk, *Fellow, IEEE, Fellow, OSA*

***Abstract*—We introduce a fully deterministic computationally efficient method for characterizing the effect of nonlinearity in optical fiber transmission systems that utilize wavelength division multiplexing and return-to-zero (RZ) modulation. The method is based on a probability analysis of the bit patterns, and it accurately accounts for bit-pattern-dependence effect due to both nonlinearly induced amplitude and timing jitter. We apply this method in calculating the error probability in a prototypical multichannel RZ undersea system in the presence of both nonlinear distortion and amplifier noise.**

***Index Terms*—Communication system nonlinearities, communication system performance, error analysis, optical fiber communication, optical propagation in nonlinear media.**

I. Introduction

NONLINEAR effects in optical fibers cause distortion of optical signals in fiber communication lines, thus imposing an upper limit on the signal power [1]–[4]. From a system designer's prospective, the nonlinear effects lead to a power penalty, which can be partially overcome by reducing the input power to the system. Thus, nearly all modern systems operate at powers at which the signal evolution is almost linear [5]. However, there always exist small nonlinear interactions, leading to a small signal distortion that accumulates during transmission over long distances and can introduce a significant system penalty [6]–[9]. Calculating the bit error ratio (BER) in long-haul systems and finding the optimal power level requires an accurate model of the nonlinear interactions.

The main challenge in characterizing the nonlinear penalty is that it is a statistical quantity. The amount of distortion that an optical pulse suffers depends on the particular pattern of surrounding pulses, which is effectively random because these pulses represent the information bits, and the information sequence of bits is quasi-random. This effect is often referred to as the nonlinear pattern-dependence effect [1], [10]. In single-channel transmission, dispersion leads to the spread of optical signals, causing approximately three to seven adjacent pulses to interfere [5], [11]. Therefore, a common approach to account for pattern-dependent nonlinear effects is to use a pseudorandom sequence of bits, which is typically 2^3 to 2^7 bits, to find the worst case bit in the sequence. When we consider a multichannel system, this approach is inappropriate since there are many more pulses interacting with each other due to the dispersive walkoff between the frequency channels. As an illustration, Fig. 1 shows three simulated optical eye diagrams of a noise-free signal in the center channel of a 10-Gb/s wavelength-division-multiplexed (WDM) return-to-zero (RZ) system after propagating over 5000 km. We used nine copolarized channels spaced by 50 GHz, and the average power was approximately -0.7 dBm per channel. We used three different sets of bit patterns in different WDM channels, while the bit pattern in the center channel remained unchanged. As we move from Fig. 1(a) to (c), it is apparent that the eye changes from being almost completely open to completely closed, which corresponds to (a) the best, (b) the intermediate, and (c) the worst bit patterns, respectively.

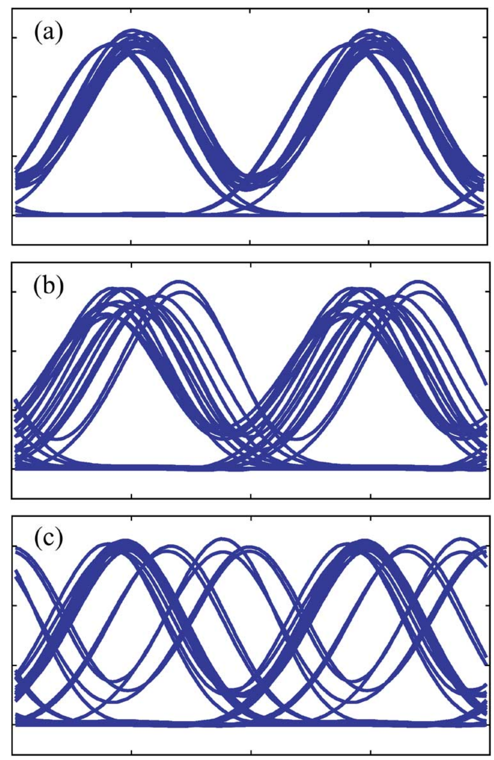

Fig. 1. Pattern dependent nonlinear effect in a WDM RZ system.

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The authors are with the University of Maryland Baltimore County, Baltimore, MD 21250 USA (e-mail: oleg.sinkin@umbc.edu; v-grigoryan@umbc.edu; menyuk@umbc.edu).

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In this case, finding the worst-case performance becomes not only prohibitively time consuming, since the number of possible interaction patterns grows exponentially, but it is also not useful because the likelihood of the worst-case pattern is negligibly small. Therefore, a probabilistic approach is necessary to treat this problem. One can use biased Monte Carlo simulations in estimating the probability density function (pdf) with the required accuracy. Monte Carlo simulations are, however, time consuming, and even with modern-day computers, it is computationally expensive to obtain an estimate of the pdf over many orders of magnitude, even with biasing. Hence, there is a need for deterministic computationally efficient techniques.

Typically, the dominant nonlinear effect in modern high-speed systems operating at 10 GB/s is cross-phase modulation (XPM) [7], [8], [12]–[15]. The manifestation of this effect depends on the light modulation format. In non-return-to-zero (NRZ) transmission, the signal distortion appears in the form of amplitude jitter [14], [16]–[18], while in the RZ systems, the dominant nonlinear effect is typically collision-induced timing jitter [12], [19]–[22]. This fact requires the development of completely different approaches to account for the nonlinear effects in these two types of systems. Because the NRZ modulation format has been the dominant modulation format for many years, techniques have been developed to characterize the nonlinear effects in WDM NRZ transmission [14], [16]–[18], [23]–[31], and the BER calculations based on these techniques agree well with experimental results. The basic idea in these approaches [16], [25]–[27] is to utilize a pump-probe method, in which the XPM-induced distortion is treated as an additive perturbation. An exception is [31], where the authors treat the distortion as multiplicative. In order to determine the influence of the nonlinearity on the system performance, one further assumes that the XPM-induced distortion may be treated as additive Gaussian noise, and a correction to the Q-factor is calculated [14], [16], [25], [28]–[30], [32].

It has been discovered that the RZ-modulated signal undergoes less intersymbol interference in the receiver and is more robust to fiber nonlinearities and, thus, more appropriate for use in long-haul data transmission [11], [21], [33]–[36]. The major nonlinear effect in WDM RZ systems is collision-induced timing jitter. Therefore, the pump-probe approach just described cannot be directly applied to the RZ systems. Collision-induced timing jitter has, however, been extensively studied in both soliton and linear systems [12], [19], [20], [37]–[43]. It is known how to calculate the time shift that results from a collision of a pair of pulses and to calculate the standard deviation of the time shift. However, no accurate BER calculation that takes into account the interchannel nonlinear bit-pattern effect due to this timing jitter has been reported in the literature prior to [22] and [44].

In our previous work [22], [44], we showed how to accurately account for the nonlinear distortion in WDM RZ systems. This result is for the case when the channel that we monitor *the target channel* contains only a single pulse, while all the neighboring channels contain arbitrary sequences of pulses. This model is based on the calculation of the pdf of collision-induced time shift, and it neglects the nonlinearly induced amplitude jitter. As we showed before, this reduced model is in excellent agreement with a full numerical model based on multicanonical Monte Carlo and standard importance-sampled (IS) Monte Carlo simulations. We use the term standard IS to refer to nonadaptive IS, as shown in the Appendix.

In this paper, we show that considering just one pulse in the target channel is not sufficient to accurately calculate the BER of a WDM system since in the presence of multiple pulses in the target channel, the combination of intra- and interchannel nonlinear interactions leads to an additional amplitude jitter that cannot be neglected. We show how to calculate the pdf of the nonlinearly induced amplitude variation of the received current that is not due to the timing jitter. We validate this method using the full statistical model based on the IS technique. Using the knowledge of the pdf of the noise-induced amplitude variation, we combine the amplified spontaneous emission (ASE) noise and nonlinear contributions to calculate the resulting BER.

II. Collision-Induced Timing Jitter

In this section, we review the reduced probabilistic model of nonlinear penalties due to the collision-induced timing jitter [22], [44].

A. Calculation of Collision-Induced Time Shift

In order to obtain the pdf of the collision-induced time shift and the BER, we start by calculating the time-shift function [40], [45]. We refer to the pulse u_T for which we are calculating the time-shift function as the target pulse and the channel in which it is located as the probe channel. The other WDM channels are referred to as the pump channels. We define the time-shift function $\tau(\Delta f_k, l) \equiv \tau_{kl}$ as the time shift of the target pulse u_T after a propagation distance L due to a collision with a pulse u_{kl} that is initially located in the lth bit slot of the kth pump channel, where Δf_k is the frequency offset of the pump channel from the probe channel. We compute this time shift using a semianalytical method presented by Grigoryan and Richter [12].

The starting point is a version of the propagation equation, in which the nonlinear part includes only the effects of self-phase modulation and interchannel XPM

$$\frac{\partial u_T}{\partial z} + i\frac{\beta''}{2}\frac{\partial^2 u_T}{\partial t^2} - i\gamma\left(|u_T|^2 + 2\sum_{k,l}\alpha_{kl}|u_{kl}|^2\right)u_T - gu_T = 0 \quad (1)$$

where z is the physical distance, t is the retarded time with respect to the probe channel, β'' is the local dispersion, γ is the nonlinear coefficient, g is the fiber loss and gain coefficient, and $\alpha_{kl} = 1$ if the lth bit slot in the kth channel contains a pulse, corresponding to a digital 1, and is zero otherwise. While including higher order dispersion poses no difficulty in principle, its effect on the system under study is negligible, and we set it to zero for simplicity. Equation (1) is derived, assuming a large channel separation and representing the field envelope in the kth channel as

$$u_k = \sum_l \alpha_{kl} u_{kl} \tag{2}$$

where u_{kl} is the field envelope of the pulse located in the lth bit slot of the kth channel. We made an assumption that pulses in one channel are well separated in time during the propagation when the nonlinearity is important so that for each channel k, we can use the approximation

$$|u_k|^2 = \left| \sum_l \alpha_{kl} u_{kl} \right|^2 \approx \sum_l \alpha_{kl} |u_{kl}|^2. \tag{3}$$

Originally, this assumption was used in the theory of collision-induced timing jitter for solitons and was well justified because in soliton communications systems, pulses in one channel do not overlap. In modern quasi-linear RZ systems, a pulse overlaps with many of its neighbors due to a large dispersive spread. However, the approximation (3) is still reasonable because the peak power of a pulse reduces when it disperses, thus reducing the nonlinear interactions. This approximation has proven to yield an accurate calculation of collision-induced timing jitter [12] and an accurate pdf of the time shift [22].

We define the central time of the target pulse u_T as $(1/E_T) \int t |u_T|^2 dt$ and its central frequency as $(1/E_T) \int \text{Im}[(\partial u_T^* / \partial t) u_T] dt$, where $E_T = \int |u_T|^2 dt$. We then use (1) to obtain the dynamic equations for collision-induced time and frequency shifts [12], [19]–[22] and the total time shift of the target pulse as

$$T_{\text{total}} = \sum_{k,l} \alpha_{kl} \tau_{kl} \tag{4}$$

where the time shift τ_{kl} is given by

$$\tau_{kl} = \int_0^L \Delta\Omega_{kl}(z) \beta''(z) dz \tag{5}$$

and $\Delta\Omega_{kl}(z)$ is the collision-induced frequency shift, whose evolution is given by

$$\frac{d\Delta\Omega_{kl}}{dz} = -\frac{2\gamma}{E_T(z)} \int_{-\infty}^{\infty} |u_T(z,t)|^2 \frac{\partial |u_{kl}(z,t)|^2}{\partial t} dt. \tag{6}$$

When higher order dispersion is zero, the pulse shape in all the channels is identical so that

$$u_{kl}(z,t) = u_T \left(z, t - \int_0^z 2\pi \Delta f_k \beta''(x) dx + T_{\text{bit}} l \right) \tag{7}$$

where T_{bit} is the bit period, and $l = 0$ corresponds to the bit slot of the target pulse.

B. *Probability Density Function of the Time Shift*

We assume that the α_{kl} are independent identically distributed random variables, each having probability 1/2 of being 1 or 0. Thus, the total shift of the target pulse T_{total} is a random variable, which is a linear combination of independent binary random variables. We compute the pdf of T_{total} using the characteristic function [22] $w(\xi)$, which is given by

$$w(\xi) = \langle \exp(i\xi T_{\text{total}}) \rangle = \left\langle \exp\left(i\xi \sum_{k,l} \alpha_{kl} \tau_{kl} \right) \right\rangle \tag{8}$$

where $\langle \cdot \rangle$ denotes the statistical average. Since the random variables α_{kl} are independent, and the probability that α_{kl} equals either 1 or 0 is 1/2, we may write

$$w(\xi) = \prod_{k,l} \left\{ \frac{1}{2} [1 + \exp(i\xi \tau_{kl})] \right\}. \tag{9}$$

Then, the pdf of the time shift is simply the Fourier transform of the characteristic function

$$p_T(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} w(\xi) \exp(-i\xi t) d\xi. \tag{10}$$

One can, in principle, accommodate more complex α_{kl}, e.g., pseudorandom, by using the appropriate characteristic function.

The number of pulse collisions is large, so that based on the central limit theorem one can assume that the distribution p_T of the time shift is Gaussian. However, as shown in [44], the Gaussian curve deviates significantly from the true pdf in the tails since the number of pulse collisions during the propagation in the system is finite and, thus, there exists a worst-case time shift.

We note that even if the number of pulse collisions becomes large in the presence of hundreds of channels, the Gaussian approximation of the time-shift pdf is still inaccurate in the tails because one of the conditions of the central limit theorem does not hold. In particular, in order for a normalized sum

$$Z_n = \frac{1}{s_n} \sum_{k=1}^{n} X_k \tag{11}$$

of independent random variables X_k with zero mean and variance σ_k^2, where

$$s_n^2 = \sum_{k=1}^{n} \sigma_k^2 \tag{12}$$

to converge to the normal pdf, it is required that for a given $\varepsilon > 0$, there exists an n such that [46]

$$\sigma_k < \varepsilon s_n, \qquad k = 1, \ldots, n. \tag{13}$$

In our case, the time shifts τ_{kl} given by (5) are inversely proportional to the square of the wavelength separation [45], or, equivalently, to the channel index k if the channel count starts from the target channel. Hence, the variance for each

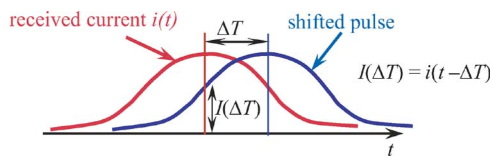

Fig. 2. Conversion of time shift to the current distortion.

individual random variable $\alpha_{kl}\tau_{kl}$ is proportional to k^4. Since the number of bits in a given channel that collide with the target pulse is proportional to k due to the dispersive walkoff, the total variance for n channels is proportional to $\sum_{k=1}^{n} 1/k^3$, which is limited by a constant for any number of channels n. Thus, (13) of the central limit theorem does not hold, and the limiting distribution of the time shift is not necessarily Gaussian. Physically, what happens, in this case, is that as k increases, the contribution of the individual bits to the timing jitter of the pulses in the target channel falls off so rapidly that it is as if on average, the target pulses were only interacting with a finite number of pulses in the other channels, even though the number really tends toward infinity and even though the worse-case timing jitter also tends logarithmically with the number of channels toward infinity.

III. Calculation of the Pulse Amplitude Distortion and the BER

In this section, we first calculate the pdf of the received current that is due to the nonlinear interactions in the absence of noise from the amplifiers. We review the model based on the timing jitter alone, and we then introduce a reduced model that accounts for the nonlinearly induced amplitude jitter in WDM systems that is not due to the timing jitter. We then validate this reduced model with IS simulations. Finally, we show how to apply our probabilistic model of nonlinear interactions to calculate the BER in the presence of amplifier noise.

A. Model of Signal Degradation Due to Collision-Induced Timing Jitter

In order to calculate the distortion of the received current signal that is due to the timing jitter, we use a simplified method, in which we calculate the pulse shape $i(t)$ at the receiver using a full propagation model based on (1) with $\alpha_{kl} = 0$ for all k and l and a receiver model that includes an optical filter, an ideal square-law photodetector, and an electrical filter. We then use this pulse shape to determine the value of the sampled current I given a time shift ΔT by using the expression

$$I(\Delta T) = i(T_0 - \Delta T) \tag{14}$$

where T_0 is the central time of the pulse, as shown in Fig. 2. We then obtain the pdf of the current using the cumulative distribution function of the time shift $F_{\Delta T}(t) = \int_{-\infty}^{t} p(\tau)d\tau$, where $p(\tau)$ is the pdf of the time shift

$$\begin{aligned} F_I(x) &= \Pr\left[i(t) < x\right] = \Pr(\Delta T < T_1 \cup \Delta T > T_2) \\ &= F_{\Delta T}(T_1) + 1 - F_{\Delta T}(T_2) \end{aligned} \tag{15}$$

where $T_1 < T_2$ are the solutions of the equation

$$i(t - T_0) = x. \tag{16}$$

The pdf of the received current $p_{I,\mathrm{TJ}}(x)$ is then given by

$$p_{I,\mathrm{TJ}}(x) = dF_I(x)/dx. \tag{17}$$

B. Multipulse Interactions and Amplitude Jitter

Up to this point, we have treated pulses of the same frequency channel as if they do not overlap. In reality, in the system under study, each pulse overlaps with a maximum of four of its neighbors in the prototypical system that we consider due to dispersive spreading. This intrachannel pulse interaction combined with interchannel nonlinear crosstalk leads to an increase in the amplitude jitter that must be accounted for in the target channel, in addition to the timing jitter, in order to obtain accurate results. Hence, we treat the electric field in the target channel as a sum of the electric fields of individual pulses rather than simply adding powers as we did previously in (3), so that

$$|u_0|^2 = \left|\sum_l \alpha_{0l} u_{0l}\right|^2. \tag{18}$$

In the other channels, we continue to neglect pulse overlap. Since in our system only five pulses in the same channel overlap due to dispersion, we consider a pseudorandom bit sequence (PRBS) of length $2^5 = 32$ bits in the target channel that contains all patterns of five bits. We then treat this PRBS as a superpulse and consider a two-body collision of this superpulse with a single pulse in a neighboring channel.

To determine the effect of the collision of the pulse in the lth bit slot of the kth channel with the superpulse, we numerically solve the nonlinear Schrödinger equation, for which the input is the superpulse in the target channel and a single pulse in the lth bit of the kth channel. We then calculate the received current $I_{kl}(t)$ in the target channel after the electrical filter. We repeat this numerical procedure for all k and l.

In the next step, we remove the time shift τ_{kl} since we previously accounted for it, and we do not want to double count it. Since pulses at the receiver are well separated, and the time shifts τ_{kl} are small, we can remove the time shifts by translating individual pulses by a corresponding time determined by the precalculated time shift function τ_{kl} in (5).

Finally, we determine the received current distortion due to the two-body collisions relative to the unperturbed solution. As an unperturbed solution, we calculate the value of the current $I_T(t)$ of a single target channel with the same PRBS of length 32, in the absence of the neighboring channels. We then obtain the current distortion δI_{kl} using the expression

$$\delta I_{kl}(t) = I_{kl}(t) - I_T(t). \tag{19}$$

We assume that for an arbitrary bit pattern, the total distortion $\delta I(t)$ can be represented as a sum of the individual contributions δI_{kl} from pairwise interactions so that

$$\delta I(t) = \sum \alpha_{kl} \delta I_{kl}(t) \tag{20}$$

where $\alpha_{kl} = 1$ or 0.

In order to obtain the pdf of the amplitude deviation $\delta I(t)$, we apply the characteristic function method, as we did for the timing jitter

$$v(\xi,t) = \langle \exp[i\xi\delta I(t)]\rangle = \left\langle \exp\left(i\xi\sum_{k,l}\alpha_{kl}\delta I_{kl}(t)\right)\right\rangle \tag{21}$$

where $\langle\cdot\rangle$ denotes the statistical average taken over the ensemble of all bit patterns. Using the independence of random variables α_{kl} and the assumption that the probability of α_{kl} is equal to either 1 or 0 is 1/2, we obtain

$$v(\xi,t) = \prod_{k,l}\frac{1}{2}\left\{1+\exp\left[i\xi\delta I_{kl}(t)\right]\right\}. \tag{22}$$

Then, the pdf of current is simply the inverse Fourier transform of the characteristic function

$$p_{\delta I}(I,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} v(\xi,t)\exp(-i\xi I)d\xi. \tag{23}$$

In order to obtain the pdf $p_{I,\mathrm{NL}}$ of the current due to nonlinear distortion, we first convolve the pdf (17) of the current at the center of the pulse and (23) due to the amplitude distortion, assuming that the two processes are independent and then average over all bits so that

$$p_{I,\mathrm{NL}}(I) = \frac{1}{N}\sum_{k=1}^{N}\int p_{I,\mathrm{TJ}}(x,t_k)p_{\delta I}(I-x,t_k)dx \tag{24}$$

where N is the number of bits, and t_k is the central time of the kth bit.

C. Validation

To validate our reduced deterministic method, we compared the results of the calculations based on (17) and (24) with the corresponding results from the full statistical model based on IS simulations. We applied our method to a prototypical undersea system with a bit rate of 10 Gb/s and a propagation distance of approximately 5000 km [22] that is similar to an experimental system reported in [47]. The transmission part includes 100 periods of a dispersion map consisting of 34 km of D_+ fiber and 17.44 km of D_- fiber followed by an amplifier. The values of dispersion, effective core area, nonlinear index, and loss are 20.17 ps/nm · km, 106.7 μm^2, 1.7×10^{-20} m^2/W, and 0.19 dB/km for the D_+ fiber and -40.8 ps/nm · km, 31.1 μm^2, 2.2×10^{-20} m^2/W, and 0.25 dB/km for the D_- fiber, respectively. The average map dispersion is -0.5 ps/nm · km, and the amount of pre- and post-compensation is 1028 and 1815 ps/nm, respectively. We used 35-ps raised-cosine pulses with a peak power of 5 mW, and we launched nine copolarized channels separated by 50 GHz, each carrying a 32-bit sequence. We verified with both a full simulation model and the reduced models described here that a further increase in the number of channels and number of bits per channel had a negligible effect on the system performance. The receiver included a 30-GHz super-Gaussian optical demultiplexer and a photodetector. We did not consider amplifier noise in this part of this paper, in which we validated our reduced model of nonlinear interactions.

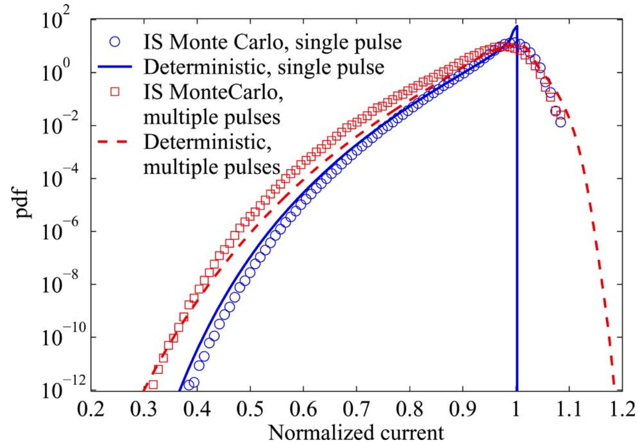

Fig. 3. Probability density function of the current at the detection point in the receiver due to the nonlinear distortion with multiple pulses in the target channel compared to a single pulse in the target channel.

To validate the proposed reduced model, we used the standard IS method [48]–[51], which we describe in detail in the Appendix. We also used the multicanonical Monte Carlo method [52]–[60] to verify that the results from the IS simulations are correct. In Fig. 3, we show the pdf $p_{I,\mathrm{NL}}(I)$ of the received current at the center of the pulse and the pdf obtained using IS simulations with multiple pulse in the target channel. For comparison, we also show the pdf $p_{I,\mathrm{TJ}}(x)$ and the pdf obtained from the corresponding IS simulations with a single pulse in the target channel.

First, we immediately notice the difference that multipulse interactions make in the low-current tail of the pdf. A deterministic model of the pdf of the current at the decision point of the receiver that only includes timing jitter alone agrees well with IS simulations when there is only a single pulse in the target channel. However, the agreement is no longer good when there are multiple pulses in the target channel. It is necessary to include nonlinearly induced amplitude jitter in the deterministic model, and we then see a good agreement between our deterministic model and IS simulation. The discrepancy in the pdf is less than an order of magnitude over the entire range of interest.

D. Calculation of the BER

Now, we show how to use the deterministic theory that we developed for the evaluation of the nonlinear penalty to calculate the BER in the presence of the ASE noise from optical amplifiers. We assume that optical noise is additive white Gaussian noise at the entry to the receiver, and we take into account the actual pulse shape, as well as the frequency-dependent optical and electrical filtering, using an approach described by Forestieri [61]. For this calculation, we used an 8-GHz electrical fifth-order Bessel filter, and we set the

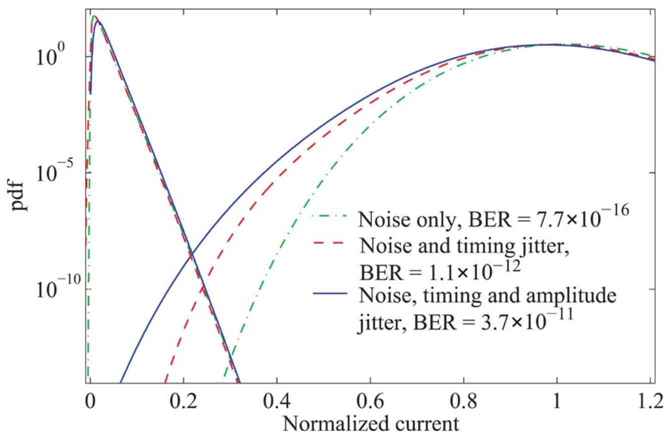

Fig. 4. PDF of the current in the marks and spaces at the decision time.

optical signal-to-noise ratio to 15 dB, calculated over a 25-GHz bandwidth. For the purpose of this paper, the choice of the noise model is not restrictive since it does not affect the methodology that we propose, and we only need the distribution of the noise-induced variation of the received current. In principle, one could also use a noise model that takes into account the nonlinear signal-noise interaction, for example, models described in [60], and [62]–[68].

We will use the assumption that the nonlinear penalty is statistically independent of the ASE noise since it greatly simplifies the analysis. In principle, these two processes are not independent, as one may envision that the noise affects the signals and that, in turn, affects the way the signals interact with each other. However, as the noise is a zero-mean process, the effect of noise will result in either a reduction or an increase of the optical power of a pulse and, consequently, of the nonlinear interference that this pulse generates. Averaging over a large statistical ensemble tends to extinguish the correlation between the noise and the nonlinear penalty. In addition, we treat the contributions of timing jitter and amplitude jitter to nonlinearly induced signal distortion as independent.

In order to compute the pdf $p(I,t)$ of the received current I at the sampling time t due to both nonlinear signal distortion and noise, we use the pdf of the collision-induced time shift $p_T(t)$ given by (10), the pdf of the nonlinearly induced amplitude distortion $p_{\delta I}(I,t)$ given by (23), and the pdf of the noise-induced distortion $p_{\text{noise}}(I,t)$. We calculate $p_{\text{noise}}(I,t)$ by propagating a single-channel signal carrying a PRBS of length 32 through the system and using an additive white Gaussian noise model [61]. The resulting pdf $p(I,t)$ is given by

$$p(I,t) = \int_{\zeta=-\infty}^{\infty} \left[\int_{\tau=-\infty}^{\infty} p_{\text{noise}}(\zeta, t-\tau) p_T(\tau) d\tau \right] \cdot p_{\delta I}(I-\zeta, t) d\zeta. \quad (25)$$

We show the results of our calculations in Fig. 4. The dot-dashed curves show the pdf of the current in the marks and spaces due only to noise. The collision-induced timing jitter increases the BER by over three orders of magnitude. The corresponding pdfs are shown with the dashed curves. Finally, the nonlinearly induced amplitude jitter degrades the BER by more than one order of magnitude, and the total pdfs are displayed with the solid curves. Since the nonlinear interactions can degrade the BER by many orders of magnitude, it is important to model them accurately. Furthermore, even though the collision-induced timing jitter is the dominant nonlinear effect in WDM RZ systems, it is necessary to consider the intra- and interchannel nonlinearly induced amplitude distortion as it leads to a further performance degradation.

IV. Conclusion

We have presented a reduced deterministic approach in calculating the BER in WDM RZ systems due to nonlinear distortion and ASE noise. The method is based on finding the complete pdf of the nonlinearly induced penalty, and it accounts for both nonlinearly induced amplitude and timing jitter.

First, we evaluated the pdf of the received current due to nonlinear effects, neglecting noise. We used the pdf of the time shift to obtain the pdf of the received current at the detection point using a pulse shape at the receiver [22]. With a single pulse in the target channel, the agreement between the reduced deterministic and full statistical approaches is excellent [44]. This agreement demonstrates that the collision-induced timing jitter is the dominant nonlinear effect in this type of system.

Then, we showed that when considering multiple pulses in the target channel, we must also account for the amplitude jitter that is induced by the inter- and intrachannel nonlinear interactions, which does not arise due to timing jitter. To calculate the pdf of the current due to this additional amplitude jitter, we applied an approach based on pairwise interactions, similar to what we did for the timing jitter. In this case, we considered the interaction of a pseudorandom pulse sequence (a superpulse) in the target channel with single pulses in the neighboring channel. Assuming additivity of the pulse distortions at the receiver, we have calculated the pdf of the current using the characteristic function approach. Then, we combined the pdfs of the current that are due to collision-induced timing jitter and nonlinearly induced amplitude jitter, assuming the statistical independence of the two. Despite the approximations, we have achieved agreement to within an order of magnitude between this reduced deterministic approach and a full statistical approach over the entire range of the pdfs that we calculated.

Finally, we showed how to calculate the BER in the presence of both nonlinear signal distortion and ASE noise. As we showed, the nonlinear effects can significantly degrade the system performance, and in order to take these effects into account, one must use a probabilistic approach.

Appendix
Biasing for IS Monte Carlo Simulations

The IS method is well known in statistics and has been used to model a wide variety of systems, including optical communications systems [48]–[51], [69]–[71]. Therefore, we will only describe the basic approach of IS and the specific features of the

IS method that we used in this paper—in particular, the choice of biasing distributions.

Suppose one has a random vector $\mathbf{z} = (z_1, z_2, \ldots, z_n)$, which is defined on a sample space Ω. In our case, the vector $\mathbf{z}$ is discrete, and it represents the set of bits in all the channels $z_j = 0$ or 1, where $j = 1, \ldots, n$. Note that one can establish a one-to-one mapping between z_j and α_{kl}. Our goal is to compute the average

$$P = E\left[f(\mathbf{z})\right] = \sum_{\Omega} f(\mathbf{z}_k) p(\mathbf{z}_k) \tag{26}$$

where $E(\cdot)$ denotes the statistical average with respect to the probability mass function $p(\mathbf{z})$, and $f(\mathbf{z})$ is an indicator function that is equal to 1 if some quantity $X(\mathbf{z})$ falls into a desired subspace Γ of Ω

$$f(\mathbf{z}) = I_\Gamma\left[X(\mathbf{z})\right]. \tag{27}$$

In our studies, the quantity $X(\mathbf{z})$ is either the collision-induced time shift or the received current. As mentioned earlier, calculating (26) exactly is not possible since $f(\mathbf{z})$ is a complicated nonlinear function of the signal and system parameters. We estimate (26) in a set of Monte Carlo simulations

$$\hat{P} = \frac{1}{M} \sum_{m=1}^{M} f(\mathbf{z}_m) \tag{28}$$

where M is the number of Monte Carlo samples. In a standard Monte Carlo simulation, one draws samples $\mathbf{z}_m$ from a distribution $p(\mathbf{z})$. In this case, the estimator $\hat{P}$ is just the number of samples in the subspace of interest S divided by the total number of samples. It follows from (28) that $E\hat{P} = P$. Typically, we are interested in rare events, for example, large nonlinear distortions that lead to transmission errors, and a standard Monte Carlo simulation would require us to use a prohibitively large number of samples to observe the small number of events in the region of interest in the sample space that leads to errors. In standard IS, we use a modified probability distribution $p^*(\mathbf{z})$ to draw samples

$$P = \sum_{\Omega} f(\mathbf{z}_k) \frac{p(\mathbf{z}_k)}{p^*(\mathbf{z}_k)} p^*(\mathbf{z}_k) = E^*\left[f(\mathbf{z}_k)\frac{p(\mathbf{z}_k)}{p^*(\mathbf{z}_k)}\right]. \tag{29}$$

The function

$$L(\mathbf{z}) = \frac{p(\mathbf{z})}{p^*(\mathbf{z})} \tag{30}$$

is called the likelihood ratio, and the distribution $p^*(\mathbf{z})$ is called the biasing distribution. Then, the estimator of P, which is denoted $\hat{P}^*$, becomes

$$\hat{P}^* = \frac{1}{M} \sum_{m=1}^{M} f(\mathbf{z}_m) L(\mathbf{z}_m) \tag{31}$$

where the samples are now drawn from the biasing distribution. Intuitively, we can sample the region of interest Γ in the sample space more efficiently if we choose the biasing distribution such that $p^*(\mathbf{z}) > p(\mathbf{z})$. Then, the number of samples falling into Γ will be larger.

Next, we describe how we choose the biasing distribution for our simulations. We are interested in the pdf of the received current. Since the dominant nonlinear effect is collision-induced timing jitter, the large signal distortion is correlated with large time shifts. Therefore, we bias the distribution of the α_{kl} toward large values of the collision-induced time shift, using our knowledge of the time shift function τ_{kl}.

Suppose we have a set of random variables x_m with the distribution

$$\begin{aligned} p(x_m = \sigma_m) &= \frac{1 + p_m}{2} \\ p(x_m = -\sigma_m) &= \frac{1 - p_m}{2}. \end{aligned} \tag{32}$$

We want to bias the distributions of x_m in such a way that the sum $z = \sum_m x_m$ has a mean near a desired value z_0. For the sake of simplicity, let us assume that z is a random variable that is approximately Gaussian-distributed with the mean z_0 and standard deviation S

$$\begin{aligned} p(z) &= \frac{1}{\sqrt{2\pi S}} \exp\left[-\frac{(z - z_0)^2}{2S}\right] \\ S &= \sum_m \sigma_m^2. \end{aligned} \tag{33}$$

We then use Bayes rule

$$p(x_m | x) = \frac{p(x_m) p(x - x_m)}{p(x)} \tag{34}$$

so that

$$p(x_m = \sigma_m | z) = \frac{g_+}{g_+ + g_-} \tag{35}$$

where

$$g_\pm = \frac{1}{\sqrt{2\pi\left(S - \sigma_m^2\right)}} \exp\left[-\frac{(z \mp \sigma_m)^2}{2\left(S - \sigma_m^2\right)}\right]. \tag{36}$$

Then, the expected value of x_m is given by

$$\begin{aligned} E(x_m) &= \sigma_m p(x_m = \sigma_m | z) - \sigma_m p(x_m = -\sigma_m | z) \\ &= \sigma_m \tanh\left(\frac{\sigma_m z}{S - \sigma_m^2}\right). \end{aligned} \tag{37}$$

On the other hand, from (32), we have

$$E(x_m) = \sigma_m \frac{1 + p_m}{2} - \sigma_m \frac{1 - p_m}{2} = \sigma_m p_m. \tag{38}$$

From (37) and (38), it follows that

$$p_m = \tanh\left(\frac{\sigma_m z}{S - \sigma_m^2}\right). \tag{39}$$

Now, we transform the set of variables x_m to the set $\alpha_{kl}\tau_{kl}$ using the mapping

$$\beta_{kl} = 2\alpha_{kl} - 1$$
$$x_m = \frac{1}{2}\beta_{kl}\tau_{kl}. \tag{40}$$

The biasing probability mass function for α_{kl} becomes

$$p^*(\alpha_{kl} = 1) = \frac{1 + p_{kl}}{2}$$
$$p^*(\alpha_{kl} = 0) = \frac{1 - p_{kl}}{2} \tag{41}$$

where

$$p_{kl} = \tanh\left(\frac{\frac{1}{2}\tau_{kl}T_{\rm goal}}{\frac{1}{4}\sum_{k,l}\tau_{kl}^2 - \tau_{kl}^2}\right) \tag{42}$$

and $T_{\rm goal}$ is the target value of the mean of the total time shift.

Despite the use of approximation (33) that the total time shift is Gaussian distributed, our simulations show that the choice of biases (42) is efficient for estimating the pdfs of both the collision-induced time shift and the received current.

In order to sample the entire pdf efficiently, we used multiple importance sampling with the balance heuristics [51], [71]. We used the values of $T_{\rm goal}$ in (42) from -50 to 50 ps with a 5-ps interval. We calculated the error of the pdf estimate from IS [51], [71], and it did not exceed 10% for all values of the time shift.

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Oleg V. Sinkin (S'01–M'07) was born in Protvino, Russia, in 1976. He received the M.S. degree in applied physics and mathematics from Moscow Institute of Physics and Technology, Moscow, Russia, in 1999 and the Ph.D. degree in electrical engineering from the University of Maryland Baltimore County, Baltimore, in 2006.

During 1996–1999, he also worked as a Research Engineer with IRE-POLUS, Moscow (IPG Photonics). His areas of expertise include fiber lasers and amplifiers and modeling of optical fiber communication systems.

Dr. Sinkin is currently a Senior Member of System Research and Development, Tyco Telecommunications.

Vladimir S. Grigoryan received the Ph.D. degree in radio physics, including quantum radio physics, from the Institute of Radio Engineering and Electronics, Russian Academy of Sciences, Moscow, Russia, in 1981.

In 1981, he was with the Yerevan Physics Institute, Yerevan, Armenia, where he worked on modeling of acceleration of neutral atoms using laser radiation, and from 1984 to 1992, he was a Senior Research Scientist with Laser Technique Corporation, Yerevan, working on computer modeling of coherent two-photon resonant interactions. From 1992 to 1996, he worked as a Lead Engineer with IRE-POLUS Corporation, Moscow (currently IPG Photonics, Oxford, MA), on modeling and design of picosecond fiber lasers. He was with the University of Maryland Baltimore County (UMBC), Baltimore, in 1996, working on advanced modeling of fiber optic communications systems as a Research Associate Professor. From 2000 to 2002, he worked as a Group Leader with the System Design and Modeling Group, Corvis Corporation, Columbia, MD. He returned to academia in 2003 as a Research Professor with the Electrical Engineering and Computer Science Department, Northwestern University, Evanston, IL, and as a Research Professor with the Computer Science and Electrical Engineering Department, UMBC, working on nonlinear effects in optical fibers and integrated semiconductor structures and quantum photonics.

Dr. Grigoryan is a member of the Optical Society of America.

Curtis R. Menyuk (SM'88–F'98) was born on March 26, 1954. He received the B.S. and M.S. degrees from the Massachusetts Institute of Technology, Cambridge, in 1976 and the Ph.D. degree from the University of California, Los Angeles, in 1981.

He has worked as a Research Associate with the University of Maryland, College Park, and with the Science Applications International Corporation, McLean, VA. In 1986, he became an Associate Professor with the Department of Electrical Engineering, University of Maryland Baltimore County (UMBC), Baltimore, where he was the founding member of the department. In 1993, he was promoted to Professor. He was on partial leave from UMBC from fall 1996 to fall 2002. From 1996–2001, he worked part time for the Department of Defense (DoD), codirecting the optical networking program with the DoD Laboratory for Telecommunications Sciences, Adelphi, MD, from 1999 to 2001. During 2001–2002, he was Chief Scientist with PhotonEx Corporation. For the last 18 years, his primary research area has been theoretical and computational studies of lasers, nonlinear optics, and fiber-optic communications. He has authored or coauthored more than 200 archival journal publications, as well as numerous other publications and presentations. He has also edited three books. The equations and algorithms that he and his research group at UMBC have developed to model optical fiber systems are used extensively in the telecommunications and photonics industry.

Dr. Menyuk is a member of the Society for Industrial and Applied Mathematics and the American Physical Society. He is a Fellow of the Optical Society of America. He is a former UMBC Presidential Research Professor.