Modeling phase noise in high-power photodetectors

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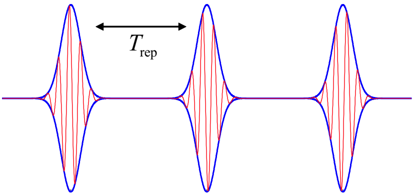
Background

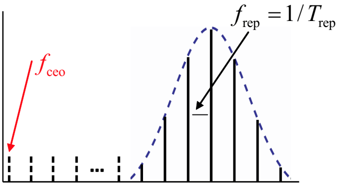
In 1999–2000: Frequency combs were invented

> *"The excitement surrounding the rapid evolution in these fields since 1999 gives us a hint of what it must have been like after 1927 when the first ideas of quantum mechanics were introduced..."*
>
> — *J. L. Hall and T. W. Hänsch, 2005 Nobel prize winners in Physics*

Time Domain



Frequency Domain



The key advance was electronically locking f_{ceo} and f_{rep}!

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Prior work

- Phase noise in photodetectors limits applications in
  - RF-photonics
  - time and frequency metrology
- ⋆ Quinlan et al.[1] theoretically predicted:
  - phase noise from a train of ultrashort optical pulses tends to zero as the optical pulse width tends to zero
- ⋆ Quinlan et al.[2] experimentally observed:
  - this decrease ceases once the optical pulse width becomes small compared to the electrical pulse width
- ⋆ Sun et al.[3] reproduced the experimental results using Monte Carlo simulations
- Monte Carlo simulations are too computationally slow to be used for performance optimization and physical insight is lost

[1] F. Quinlan et al., J. Opt. Soc. Am. B **30**, 1775–1785 (2013).
[2] F. Quinlan et al., Nat. Photonics **7**, 290–293 (2013).
[3] W. Sun et al., Phys. Rev. Lett. **113**, 203901 (2014).

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This Work

- We use the drift-diffusion equations instead of Monte Carlo simulations
  - This approach takes minutes on a desktop computer, as opposed to hours on a computer cluster
- We explain analytically that the mean-square phase noise tends to a constant non-zero value when the optical pulse width tends to zero
- We use our approach to design an optimized device

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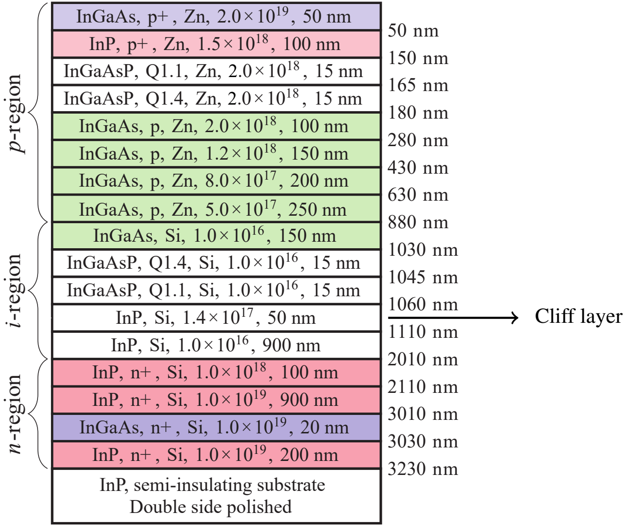
Structure of the MUTC photodetector we model



MUTC structure fabricated by Li et al.[1]

[1] Z. Li et al., IEEE J. Quantum Electron. **46**, 626–632 (2010).

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Drift-Diffusion Model

$$\frac{\partial n}{\partial t} = G_{\text{opt}} + G_{\text{ii}} - R(n,p) + \frac{\nabla \cdot \mathbf{J}_n}{q}$$

$$\frac{\partial p}{\partial t} = G_{\text{opt}} + G_{\text{ii}} - R(n,p) - \frac{\nabla \cdot \mathbf{J}_p}{q}$$

$$0 = \nabla \cdot \nabla \varphi + \frac{q}{\epsilon}\left(N_D^+ + p - n - N_A^-\right)$$

n	electron density	p	hole density
G_{opt}	optical generation rate	G_{ii}	impact ionization generation rate
R	recombination rate	φ	electric potential
$\mathbf{J}_n$	electron current density	$\mathbf{J}_p$	hole current density
N_D^+	donor density	N_A^-	acceptor density

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Drift-Diffusion Model

$$\mathbf{J}_p = qp\mathbf{v}_p(\mathbf{E}) - qD_p(\mathbf{E})\nabla p, \quad \mathbf{J}_n = qn\mathbf{v}_n(\mathbf{E}) + qD_n(\mathbf{E})\nabla n$$

$$G_{\text{ii}} = \alpha_n \frac{|\mathbf{J}_n|}{q} + \alpha_p \frac{|\mathbf{J}_p|}{q}, \quad G_{\text{opt}} = G_c \exp\left[-\alpha(L-x)\right]$$

$$G_c = Q\alpha, \quad Q = \frac{P}{A}$$

$\mathbf{v}_n$	electron drift velocity	$\mathbf{v}_p$	hole drift velocity
D_n	electron diffusion coefficient	D_p	hole diffusion coefficient
G_c	generation rate coefficient	α	absorption coefficient
L	device length	x	distance across the device
A	the area of the light beam	P	the power of the light beam
α_n	electron impact ionization coefficient	α_p	hole impact ionization coefficient

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Drift velocity model

Empirical expressions that have been used to fit $\mathbf{v}_n(\mathbf{E})$ for electrons[1] and $\mathbf{v}_p(\mathbf{E})$ for the holes[2] are given by

$$\mathbf{v}_n(\mathbf{E}) = \frac{\mathbf{E}\left(\mu_n + v_{n,\mathrm{sat}}\beta|\mathbf{E}|\right)}{1 + \beta|\mathbf{E}|^2}, \qquad \mathbf{v}_p(\mathbf{E}) = \frac{\mu_p v_{p,\mathrm{sat}}\mathbf{E}}{\left(v_{p,\mathrm{sat}}^{\gamma} + \mu_p^{\gamma}|\mathbf{E}|^{\gamma}\right)^{1/\gamma}}$$

μ_n	:	low-field electron mobility	μ_p	:	low-field hole mobility
$v_{n,\mathrm{sat}}$	:	saturated electron velocity	$v_{p,\mathrm{sat}}$	:	saturated hole velocity
β	:	fitting parameter	γ	:	fitting parameter

[1] M. Dentan and B. de Cremoux, J. Lightw. Technol. **8**, 1137–1144 (1990).

[2] K. W. Böer, *Survey of Semiconductor Physics* (Van Nostrand Reinhold, 1990).

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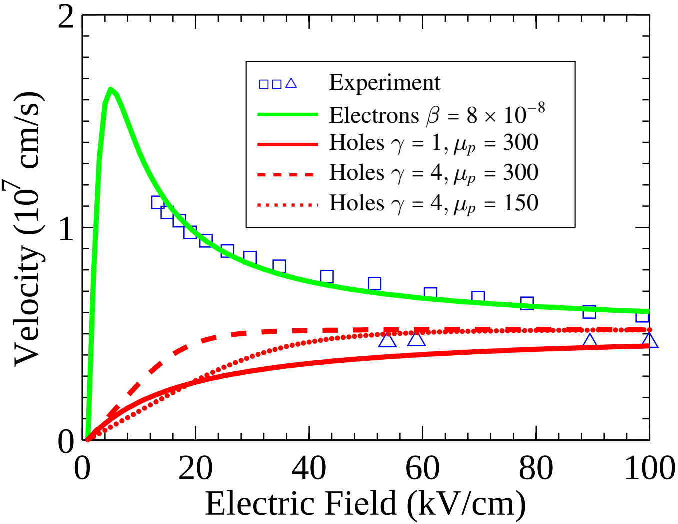
Velocity of electrons and holes in InGaAs



[1] T. H. Windhorn et al., J. Electron. Mater. **11**, 1065–1082 (1982).

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Diffusion model

Empirical expressions that have been used to fit $D_n(\mathbf{E})$ for electrons[1] and $D_p(\mathbf{E})$ for the holes[1] are given by

$$D_n(\mathbf{E}) = \frac{k_B T \mu_n / q}{\left[1 - 2\left(|\mathbf{E}|/E_p\right)^2 + \frac{4}{3}\left(|\mathbf{E}|/E_p\right)^3\right]^{1/4}}, \quad D_p(\mathbf{E}) = \frac{k_B T}{q}\frac{\mathbf{v}_p(\mathbf{E})}{\mathbf{E}}$$

E_p : fitting parameter, 4×10^3 V/cm

[1] K. J. Williams, "Microwave nonlinearities in photodiodes," PhD Dissertation, University of Maryland College Park, Maryland, USA, 1994.

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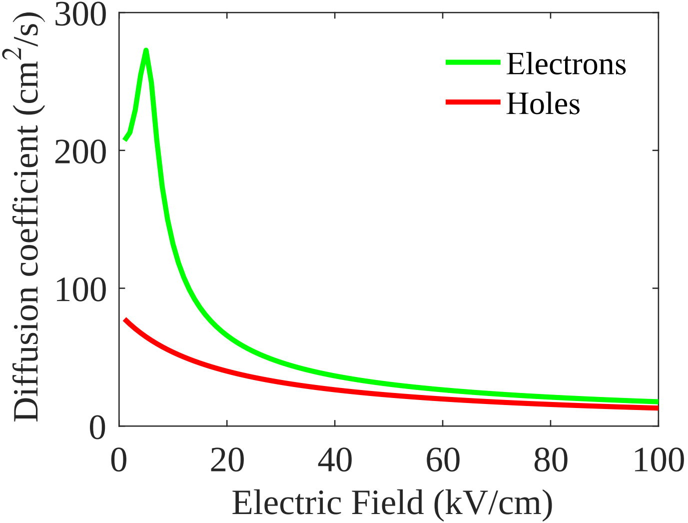
Diffusion model



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Recombination-Generation

- The largest contribution to recombination is the Shockley-Read-Hall (SRH) effect.
  - also known as trap-assisted nonradiative recombination
  - the expression for SRH recombination is

$$R = \frac{np - n_i^2}{\tau_p(n + n_i) + \tau_n(p + n_i)}$$

τ_n	:	electron lifetime	τ_p	:	hole lifetime
n_i	:	intrinsic doping concentration			

- The generation rate from impact ionization:

$$G_{\text{ii}} = \alpha_n \frac{|\mathbf{J}_n|}{q} + \alpha_p \frac{|\mathbf{J}_p|}{q}, \qquad \alpha_{n,p} = A_{n,p} \cdot \exp\left[-\left(\frac{B_{n,p}}{|\mathbf{E}|}\right)^m\right]$$

A_n, A_p, B_n, B_p, m : impact ionization parameters

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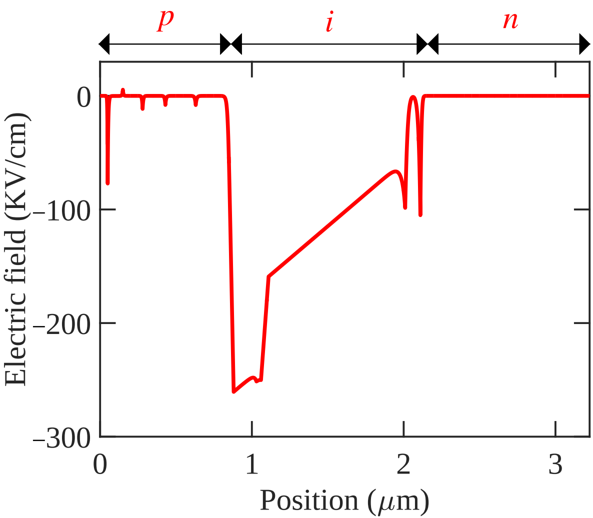
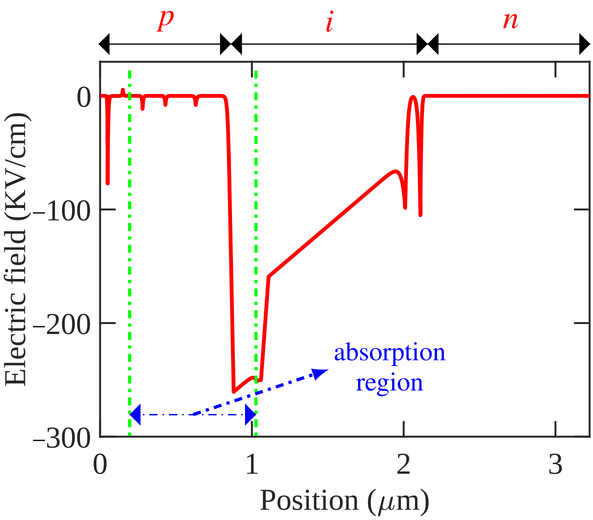
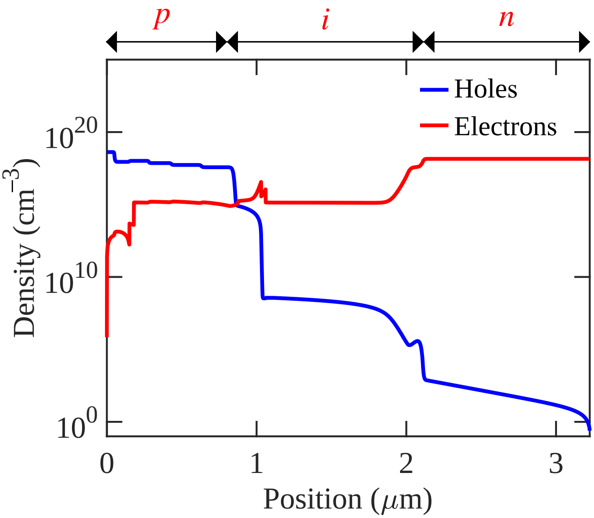
Electric field inside the MUTC photodetector



Electric field inside the MUTC photodetector



Carrier concentration inside the MUTC photodetector



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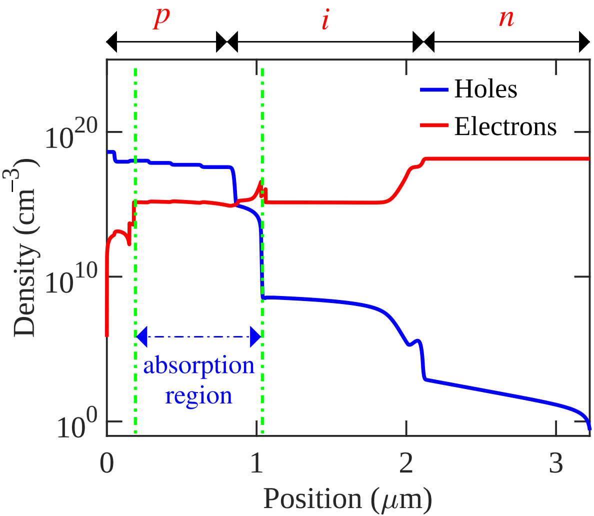
Carrier concentration inside the MUTC photodetector



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Calculation of the impulse response

- To calculate the impulse response,
  - we first calculate the steady state output current
  - we then perturb the optical generation rate and calculate the perturbed current
- The perturbed optical generation rate ΔG_{opt} is defined as

$$\Delta G_{\text{opt}} = r G_{\text{opt}} \operatorname{sech}\left(\frac{t}{\tau}\right)$$

  - r = perturbation coefficient
  - t = time
  - τ = impulse width
  - $\operatorname{sech}(x)$ = hyperbolic secant function

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Calculation of the impulse response

- We define the normalized impulse response $h(t)$ as

$$h(t) = \frac{\Delta I_{\text{out}}(t)}{\int_0^\infty \Delta I_{\text{out}}(t)\,\mathrm{d}t}$$

  - $\Delta I_{\text{out}}(t)$ = change in the output current due to the perturbed optical generation rate

- The transfer function $H(f)$ of the photodetector is defined as

$$H(f) = \int_{-\infty}^{\infty} h(t)\exp(-2\pi jft)\,\mathrm{d}t$$

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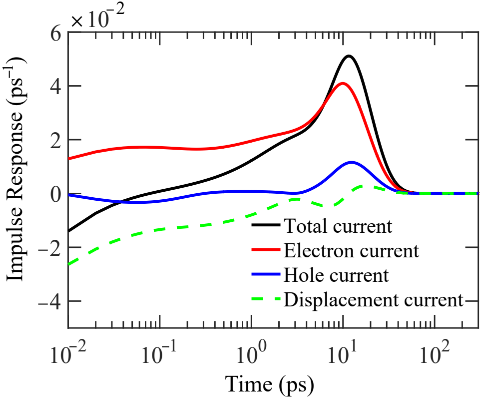
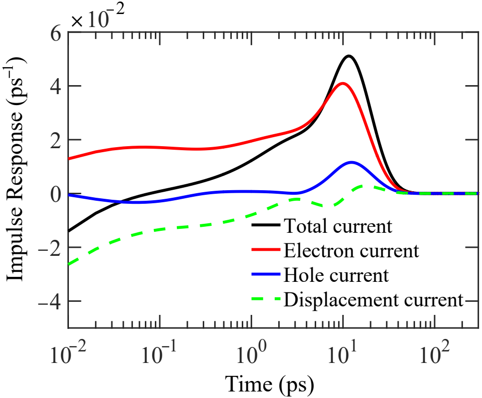
Impulse response of the MUTC photodetector



Impulse response of the MUTC photodetector



- The displacement current dominates the total current for the first 50 fs

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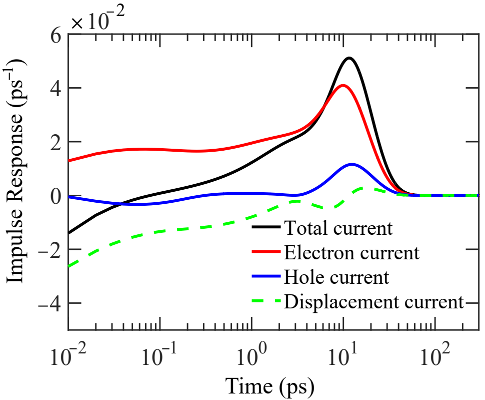
Impulse response of the MUTC photodetector



- The displacement current dominates the total current for the first 50 fs
- Thereafter, the electron current dominates at all times

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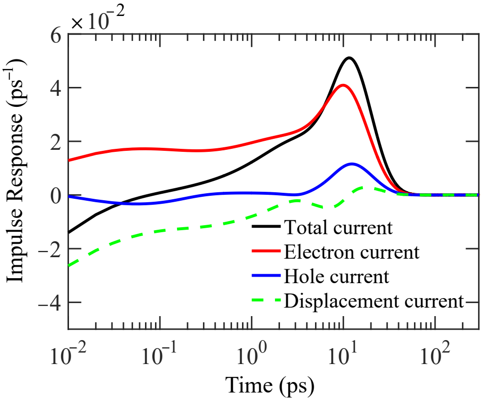
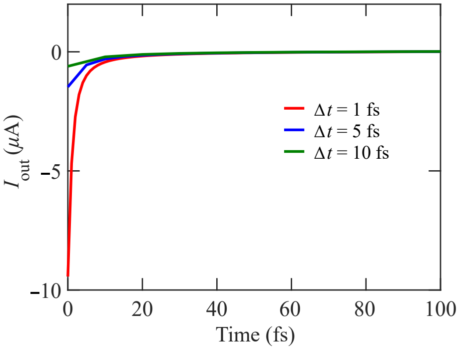
Impulse response of the MUTC photodetector



- The displacement current dominates the total current for the first 50 fs
- Thereafter, the electron current dominates at all times
- Hole current does not play a major role

Calculation of the impulse response



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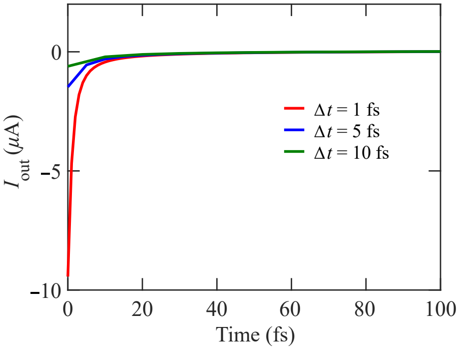
Calculation of the impulse response



- We compared three different time meshes Δt

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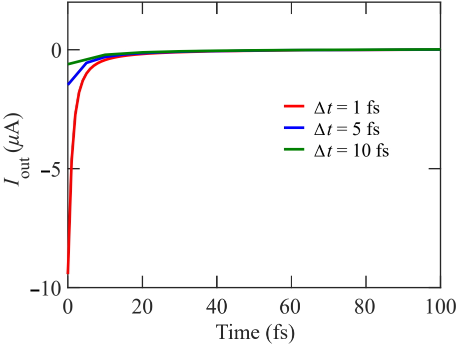

- We compared three different time meshes Δt
- The results are almost identical for $t > 20$ fs

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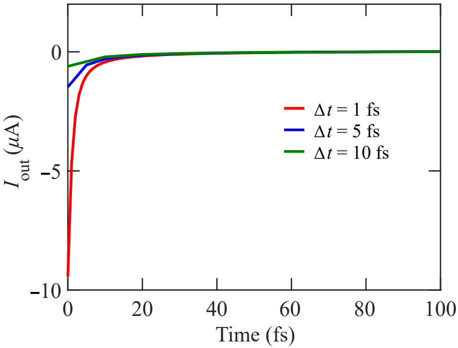

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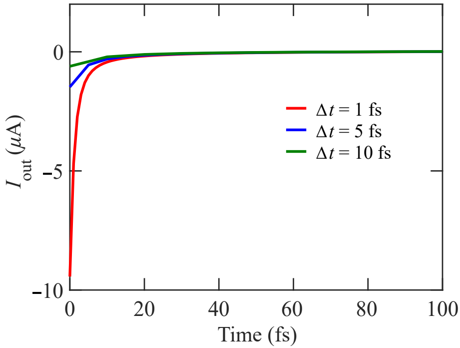

- We compared three different time meshes Δt
- The results are almost identical for $t > 20$ fs
- The frequency dependence is reliable up to frequencies of 50 THz
  - far beyond the device limit of 10–50 GHz

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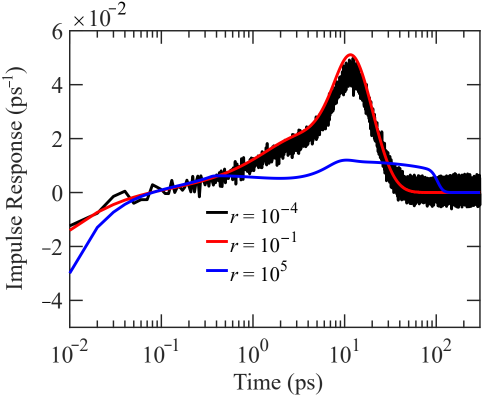
Calculation of the impulse response



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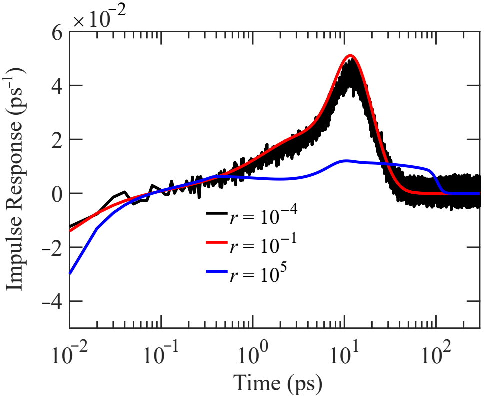
Calculation of the impulse response



- When $r = 10^{-4}$, computational errors degrade the impulse response

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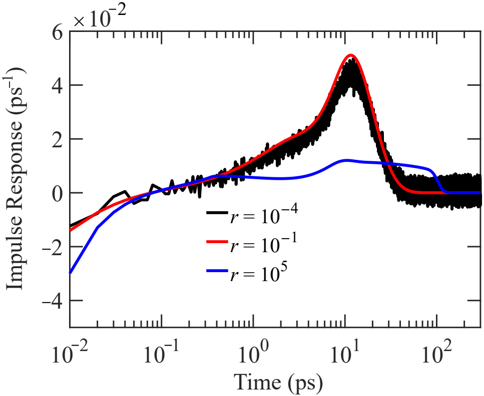
Calculation of the impulse response



- When $r = 10^{-4}$, computational errors degrade the impulse response
- When $r = 10^{5}$, nonlinearity distorts the impulse response

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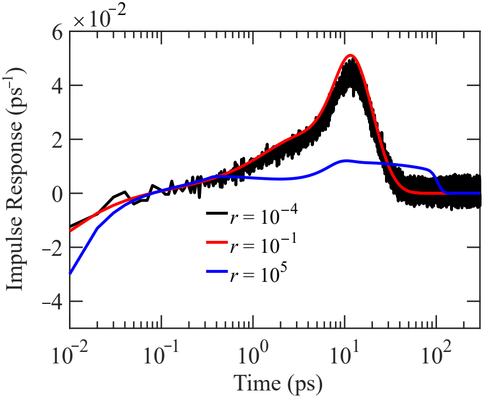
Calculation of the impulse response



- When $r = 10^{-4}$, computational errors degrade the impulse response
- When $r = 10^{5}$, nonlinearity distorts the impulse response
- For $10^{-3} < r < 10^{4}$, the impulse response is almost identical

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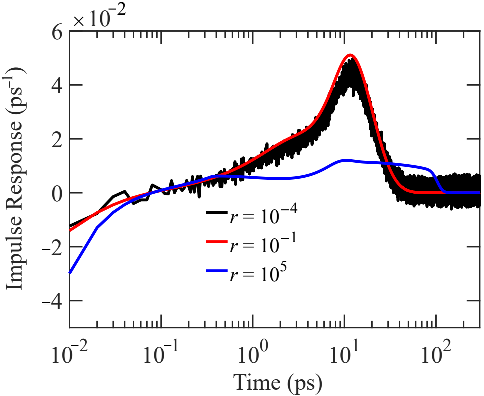
Calculation of the impulse response



- When $r = 10^{-4}$, computational errors degrade the impulse response
- When $r = 10^{5}$, nonlinearity distorts the impulse response
- For $10^{-3} < r < 10^{4}$, the impulse response is almost identical
- We use $r = 10^{-1}$ in our calculations

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Calculation of the phase noise

- The mean-square phase fluctuation is given by[1]

$$\left\langle \Phi_n^2 \right\rangle = \frac{1}{N_{\text{tot}}} \frac{\int_0^{T_R} h(t) \sin^2\left[2\pi n(t - t_c)/T_R\right] \mathrm{d}t}{\left\{\int_0^{T_R} h(t) \cos\left[2\pi n(t - t_c)/T_R\right] \mathrm{d}t\right\}^2}$$

  - N_{tot} = total number of electrons in the photocurrent
  - t_c = central time of the output current
  - T_R = repetition time between optical pulses
- In the limit of short optical pulse widths ($\leqslant$ 500 fs):
  - $\langle \Phi_n^2 \rangle$ tends to a non-zero constant

$$\left\langle \Phi_n^2 \right\rangle = \frac{1}{N_{\text{tot}}} \frac{\int_0^{T_R} h_e(t) \sin^2\left[2\pi n(t - t_c)/T_R\right] \mathrm{d}t}{\left\{\int_0^{T_R} h_e(t) \cos\left[2\pi n(t - t_c)/T_R\right] \mathrm{d}t\right\}^2}$$

★ $h_e(t)$ = electronic impulse response of the device

[1] S. E. Jamali Mahabadi et al., Opt. Express **27**, 3717–3730 (2019).

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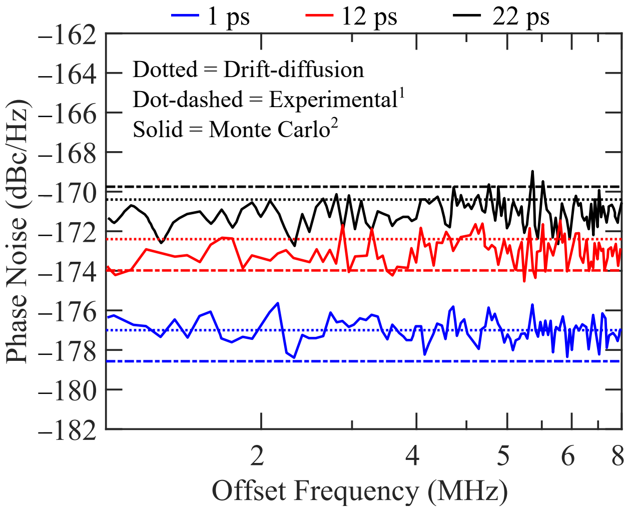
Phase noise in the MUTC photodetector



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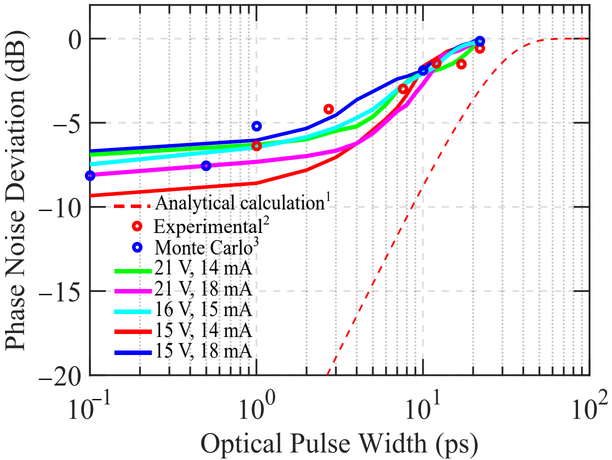

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[3] W. Sun et al., Phys. Rev. Lett. **113**, 203901 (2014).

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Conclusions

- We used the drift-diffusion equations to calculate
  - the impulse response
  - the phase noise

  in an MUTC photodetector with short optical pulses
- We found excellent agreement with prior experiments[1] and Monte Carlo simulations[2]
- Advantages of our approach
  - orders of magnitude faster than Monte Carlo simulations
  - enables device optimization
  - physical insight
- We determined the parameters for simulating photodetectors in pulse mode
  - drift and diffusion velocity coefficients
  - mesh size and time step

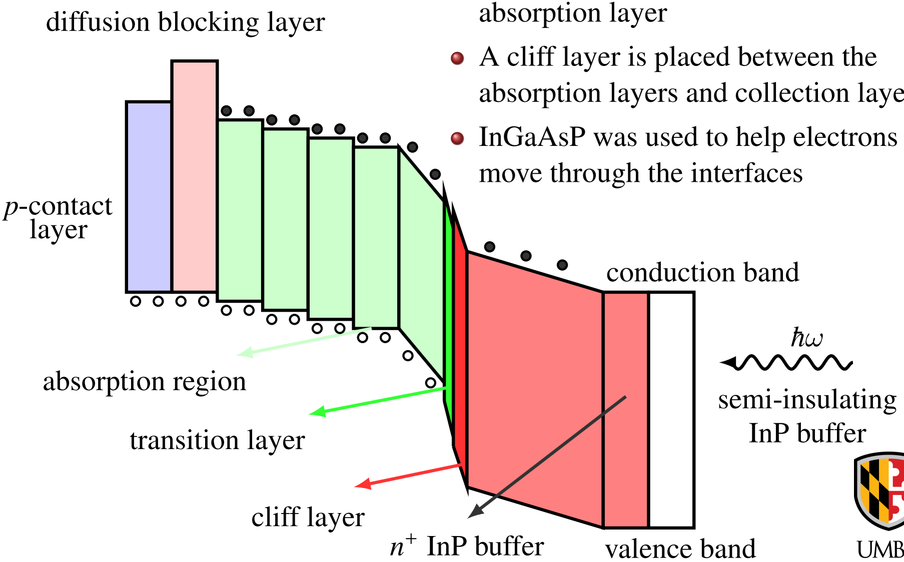
[1] F. Quinlan et al., Nat. Photonics **7**, 290–293 (2013).

[2] W. Sun et al., Phys. Rev. Lett. **113**, 203901 (2014).

Thank you for your attention

Backup slides

Structure of an MUTC photodetector



- A thin intrinsic layer was used as an absorption layer
- A cliff layer is placed between the absorption layers and collection layers
- InGaAsP was used to help electrons move through the interfaces

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Implicit method

Fully implicit method[1] (backwards Euler method) is used to solve the equations

$$\begin{aligned}\frac{n^{t+1}-n^{t}}{\Delta t} &= F_n(n^{t+1},p^{t+1},\varphi^{t+1})\\ \frac{p^{t+1}-p^{t}}{\Delta t} &= F_p(n^{t+1},p^{t+1},\varphi^{t+1})\\ 0 &= F_\varphi(n^{t+1},p^{t+1},\varphi^{t+1})\end{aligned}$$

[1] S. Selberherr, *Analysis and simulation of semiconductor devices*. New York: Springer-Verlag Wien, 1984.

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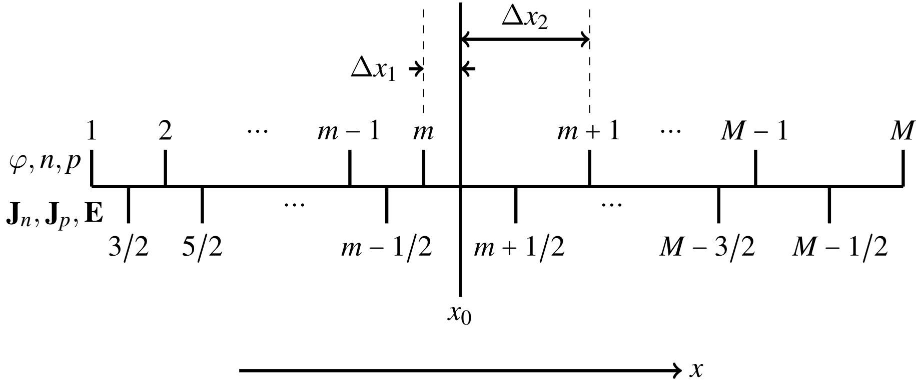
Modeling Approach (1D Scheme)



Gridding scheme used in device model for multilayer devices

- φ, n, p are defined at integer grid values
- $\mathbf{J}_n, \mathbf{J}_p, \mathbf{E}$ are defined at half-integer grid values

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Phase Noise

- We define the finite-time Fourier transform,

$$F_T\{x(t)\} \equiv \int_{-T/2}^{T/2} x(t)\exp(-j2\pi ft)\mathrm{d}t$$

- We next write

$$\begin{aligned} F_T\{i(t)\} &= \int_{-T/2}^{T/2} i(t)\exp(-j2\pi ft)\mathrm{d}t \\ &= \frac{1}{2K}\sum_{k=-K}^{K-1}\int_0^{T_R} i(t+kT_R)\exp[-j2\pi f(t+kT_R)]\mathrm{d}t \end{aligned}$$

  - $i(t)$ = output current
  - T_R = repetition time between optical pulses
  - $T = KT_R$

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Phase Noise

- If we let $i_k(t) = i(t + kT_R)$, so that $i_k(t)$ is the k-th current output pulse, we obtain

$$F_T\{i(t)\} = \frac{1}{2K}\sum_{k=-K}^{K-1}\int_0^{T_R} i_k(t)\exp(-j2\pi ft)\mathrm{d}t$$

- For the n-th harmonic of the current, we obtain

$$R_n + jQ_n = \frac{1}{2K}\sum_{k=-K}^{K-1}\int_0^{T_R} i_k(t)\left[\cos\left(2\pi nf_r t\right) - j\sin\left(2\pi nf_r t\right)\right]\mathrm{d}t$$

  - R_n = in-phase component of the n-th harmonic
  - Q_n = quadrature component of the n-th harmonic

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Phase Noise

- We also define the ensemble average $\langle c_k(t) \rangle$ for any quantity $c_k(t)$ as

$$\langle c_k(t) \rangle \equiv \lim_{K \to \infty} \frac{1}{2K} \sum_{k=-K}^{K-1} c_k(t)$$

It is useful to shift the time to remove the quadrature component to good approximation

$$R_n + jQ_n = \frac{1}{2K} \sum_{k=-K}^{K-1} \int_0^{T_R} i_k(t) \left\{ \cos\left[\frac{2\pi n}{T_R}(t - t_c)\right] - \right.$$

$$\left. j \sin\left[\frac{2\pi n}{T_R}(t - t_c)\right] \right\} \mathrm{d}t$$

  - t_c = central time of the output current

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Phase Noise

- t_c is defined by

$$Q_n = \int_0^{T_R} \langle i_k(t) \rangle \sin\left[\frac{2\pi n}{T_R}(t - t_c)\right] \mathrm{d}t = 0$$

- We define

$$\Phi_n = \frac{-j \sum_{k=-K}^{K-1} \int_0^{T_R} i_k(t) \sin\left[\frac{2\pi n}{T_R}(t - t_c)\right] \mathrm{d}t}{\sum_{k=-K}^{K-1} \int_0^{T_R} i_k(t) \cos\left[\frac{2\pi n}{T_R}(t - t_c)\right] \mathrm{d}t} = 0$$

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Phase Noise

- Although we have $\Phi_n = 0$, the separate phase contributions of each comb pulse to Φ_n will be non-zero. We have $\Phi_n = \sum_k \Phi_{kn}$ and $Q_n = \sum_k Q_{kn}$, where

$$\Phi_{kn} = \frac{Q_{kn}}{R_n} = \frac{-j\int_0^{T_R} i_k(t)\sin\left[\frac{2\pi n}{T_R}(t-t_c)\right]\mathrm{d}t}{\int_0^{T_R} i_k(t)\cos\left[\frac{2\pi n}{T_R}(t-t_c)\right]\mathrm{d}t}$$

- We find

$$\Phi_{kn}^2 = \frac{\int_0^{T_R}\int_0^{T_R} i_k(t) i_k(u)\sin\left[\frac{2\pi n}{T_R}(t-t_c)\right]\sin\left[\frac{2\pi n}{T_R}(u-t_c)\right]\mathrm{d}t\mathrm{d}u}{\left\{\int_0^{T_R} i_k(t)\cos\left[\frac{2\pi n}{T_R}(t-t_c)\right]\mathrm{d}t\right\}^2}$$

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Phase Noise

- We may assume that the electrons in each current pulse are Poisson-distributed
- This assumption may seem surprising at first since the photodetectors of interest to us operate in a nonlinear regime
- The electrons only interact through the electric field that they collectively create
- Due to the large number of electrons that create this field, a mean-field approximation is valid, and the arrival time of the electrons is nearly independent
- Given the assumption that the current pulses are Poisson-distributed, we find

$$\langle i_k(t) i_k(u) \rangle - \langle i_k(t) \rangle \langle i_k(u) \rangle = h(t) e^2 N_{\text{tot}} \delta(t-u)$$

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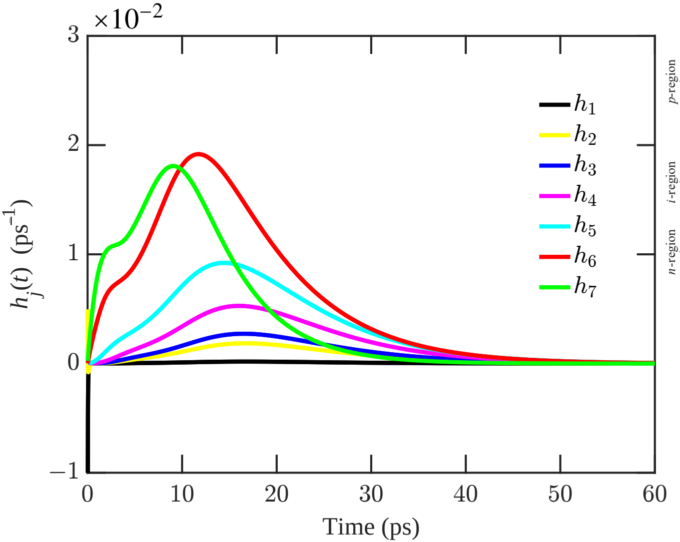
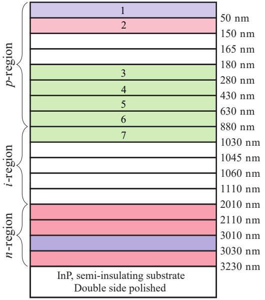
Device optimization

- Our goal is to reduce the tails in the impulse response
  - A long tail in the impulse response translates to higher phase noise

- We first altered the thickness of each absorption layer up to 10%
  - no significant change
- We next altered the doping density in each of the absorption layers
  - a smaller tail and lower phase noise

Device optimization





$$h(t) = \sum_{j=1}^{N} h_j(t),$$

Device optimization

- Phase Noise 1 = phase noise of the Li et al.[1] structure
- Phase Noise 2 = phase noise of the modified structure
- Difference = (Phase Noise 1) − (Phase Noise 2)

Pulse Width	Original structure	Modified structure	Difference
1 ps	−178.6 dBc/Hz	−180.0 dBc/Hz	1.4 dBc/Hz
12 ps	−174.0 dBc/Hz	−175.5 dBc/Hz	1.5 dBc/Hz
22 ps	−169.7 dBc/Hz	−172.8 dBc/Hz	3.1 dBc/Hz

[1] Z. Li et al., IEEE J. Quantum Electron. **46**, 626–632 (2010).

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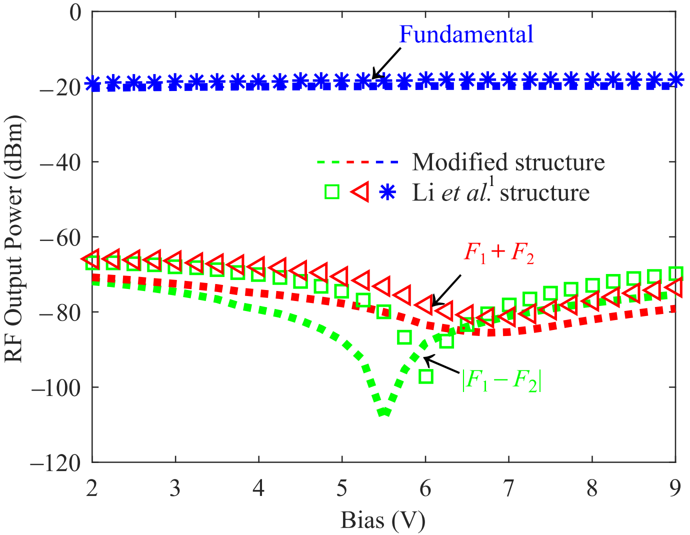
Device optimization



[1] Z. Li et al., IEEE J. Quantum Electron. **46**, 626–632 (2010).

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