Self-starting of passively mode-locked lasers with fast saturable absorbers

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Self-starting of passively mode-locked lasers with fast saturable absorption is studied. Our basic assumption is that the lasers will self-start when cw operation is unstable and mode-locked operation is stable. We start with a standard model, closely related to the Ginzburg–Landau equation, that is valid when the change in the time variation of the laser light during one round trip through the laser is small, and we determine the parameter regime in which cw mode operation becomes unstable. Coupled with previous results on the stability of mode-locked operation, these results allow us to determine when a laser will self-start. We apply our theory to figure-eight lasers with external gain.

Passively mode-locked lasers typically have two possible steady-state or equilibrium behaviors. The first is cw operation and the second is pulsed operation. Of course, a third possibility, often observed in practice, is that there is no steady state. One wants to operate mode-locked lasers in the pulsed mode and avoid cw or non-steady-state operation. It is desirable for the laser to self-start, which means that after the laser is turned on it automatically settles into pulsed operation without the need for any further modulation or adjustment of the laser. In practice, one often finds that it is necessary to modulate the laser's pump in some way to obtain mode-locked pulses.

Theoretical modeling of passively mode-locked lasers has been based on two different approaches. The first is *ab initio* simulations in which one simulates the entire evolution of light within the laser starting from noise. The second, which is analytical, is based on an idealization in which one assumes that the light evolution during one round trip through the laser is small. One then derives a simple equation, closely related to the Ginzburg–Landau equation, that describes the evolution. Finally, one determines the equilibria of this equation and their stability. This approach was first pioneered by Haus,[1] who has in recent years applied it extensively to passively mode-locked, fiber-based lasers.[2] To determine when a laser will self-start within this approach, one must find both the cw and the pulsed equilibrium solutions and determine when the cw solution is unstable while the pulsed solution is stable. Under these conditions, one assumes that the laser will self-start; this assumption underpins the theoretical work to date.[3–5] Although this analysis does not rule out the possibility that the laser could exhibit non-steady-state rather than pulsed mode operation, extensive *ab initio* simulations indicate that this assumption works well in practice.[6]

In previous work, we have already considered the stability of pulsed-mode operation.[7] In this work, we consider the stability of cw operation and determine the parameter regime in which passively mode-locked lasers with fast saturable absorption are expected to self-start. Our basic starting point is the equation

$$\frac{\partial U}{\partial z} = (g - l + i\theta)U + (B + iD)\frac{\partial^2 U}{\partial t^2} + (\Gamma + iK)|U|^2 U, \tag{1}$$

where z is the round-trip number, t is time, l and θ are the loss and phase change per round trip, B and D account for the effect of a frequency limiter and dispersion, and Γ and K account for the effect of a fast saturable absorber and nonlinearity. Our analysis, which is analogous to that of the usual modulational instability of the Schrödinger equation, resembles the earlier study of Krausz *et al.*[4] Our study differs from theirs principally in retaining a nonzero dispersion and phase change per pass. These terms contribute significantly to the behavior of fiber-based lasers, such as the ring laser and figure-eight laser, which are our principal focus. To model the gain, we use the two-level rate equation[2–7]

$$\frac{\mathrm{d}g}{\mathrm{d}t} = -\frac{g - g_0}{T_0} - \frac{|U(t)|^2}{T_0 P_{\mathrm{sat}}} g, \tag{2}$$

where T_0 is the decorrelation time and P_{sat} is the saturation power. The decorrelation time is typically much shorter than the level lifetime, and Haus and Ippen[5] have identified reflections in the laser cavity as a plausible source of the decorrelation. When $U = U_c$ is time independent, we may write the steady-state gain as $g_s = g_0/(1 + U_c^2/P_{\mathrm{sat}})$. As in Ref. 7, we assume that the bandwidth is not limited by the gain curve and we treat B as an independent parameter.

The laser equation, Eq. (1), has a cw solution $U = U_c \exp(iP_c z)$, where both U_c and P_c are real. Substituting this cw solution into Eq. (1) and using the steady-state gain g_s, we may determine the cw solution from the relations

$$\frac{g_0}{1 + U_c^2/P_{\mathrm{sat}}} = l - \Gamma U_c^2, \tag{3a}$$

$$P_c = \theta + K U_c^2. \tag{3b}$$

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In most cases there are two values of U_c satisfying Eq. (3a), but the larger one is not physically meaningful because the corresponding solution is always unstable.

We now study the stability of this cw solution. Perturbing around this solution, we set $g = g_s + \delta g$ and $U = (U_c + \tilde{u})\exp(iP_c z)$, where δg and $\tilde{u}$ are perturbations of the gain and the cw solution, respectively. From Eqs. (1) and (2), it follows that

$$\frac{\partial \tilde{u}}{\partial z} = \delta g U_c + (B + iD)\frac{\partial^2 \tilde{u}}{\partial t^2} + (\Gamma + iK)U_c^2(\tilde{u} + \tilde{u}^*), \tag{4}$$

where the asterisk stands for complex conjugation, and

$$\delta g = -\frac{\epsilon_c}{T_c}\int_{-\infty}^{t}(\tilde{u} + \tilde{u}^*)\exp\left(-\frac{t - t'}{T_c}\right)\mathrm{d}t', \tag{5}$$

where $\epsilon_c = g_0 U_c/[(1 + U_c^2/P_{\mathrm{sat}})^2 P_{\mathrm{sat}}]$ and $T_c = T_0/(1 + U_c^2/P_{\mathrm{sat}})$ is the effective decorrelation time of the cw solution.

Because of the presence of $\tilde{u}^*$, we must take into account the conjugate of Eq. (4) when determining the stability of the cw solution. Introducing a new variable $\tilde{v} = \tilde{u}^*$, which is formally independent of $\tilde{u}$, and writing $\tilde{u} = A_1 \exp(\lambda z + i\omega t)$ and $\tilde{v} = A_2 \exp(\lambda z + i\omega t)$, we obtain the eigenvalue problem

$$\lambda\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \left[\begin{pmatrix} M_1 & M_2 \\ M_2^* & M_1^* \end{pmatrix} - \frac{U_c\epsilon_c}{1 + i\omega T_c}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\right]\begin{pmatrix} A_1 \\ A_2 \end{pmatrix}, \tag{6}$$

where $M_1 = -(B + iD)\omega^2 + (\Gamma + iK)U_c^2$ and $M_2 = (\Gamma + iK)U_c^2$.

From Eq. (6) we obtain the dispersion relation

$$\lambda = \Gamma U_c^2 - B\omega^2 - \frac{U_c\epsilon_c}{1 + i\omega T_c} \pm \left[\left(\frac{U_c\epsilon_c}{1 + i\omega T_c} - \Gamma U_c^2\right)^2 - D^2\omega^4 + 2DKU_c^2\omega^2\right]^{1/2}. \tag{7}$$

There are two solutions for λ at each value of ω. When $\omega = 0$, the two solutions of λ are 0 and $2\Gamma U_c^2 - 2\epsilon_c U_c$, and when $\omega \to \pm\infty$, the solution tends toward $-B\omega^2 + \Gamma U_c^2 \pm i(D\omega^2 - KU_c^2)$. The term $U_c\epsilon_c/(1 + i\omega T_c)$ is due to gain saturation. Its effect is to stabilize cw operation because without this term the system always has positive eigenvalues λ when ω is near 0. This effect is similar to the stabilization that gain saturation provides for pulsed-mode operation.[2,7]

For the laser to self-start, λ must be positive over some range of ω, indicating that cw operation is unstable. In Fig. 1 we plot $\lambda(\omega)$ parametrically as a function of ω on the complex λ plane for two different parameter sets listed in the figure captions. Figure 1(a) corresponds to stable cw operation because $\mathrm{Re}(\lambda) < 0$ for all ω, whereas Fig. 1(b) corresponds to unstable operation because $\mathrm{Re}(\lambda) > 0$ at some values of ω. The parameters are typical for figure-eight lasers.[8,9] The model that we use, Eq. (1), is consistent with assuming that the gain is outside the Sagnac loop, as in the experiment of Wu *et al.*[9] From Eq. (7) we can extract several qualitative rules that govern the laser's ability to self-start: (1) If $\Gamma U_c > \epsilon_c$, then cw operation is not stable near $\omega = 0$ and the laser will self-start. Raising Γ aids self-starting because U_c increases while ϵ_c increases. This observation is consistent with the experiments of Goodberlet *et al.*[10] (2) Larger decorrelation time aids self-starting. The contribution from the term $\epsilon_c U_c/(1 + i\omega T_c)$ in Eq. (7) becomes smaller when T_c becomes larger. The effect is equivalent to reducing ϵ_c, which aids self-starting, as previously noted. This result is in agreement with the heuristic argument of Ref. 3 in which a smaller emission cross

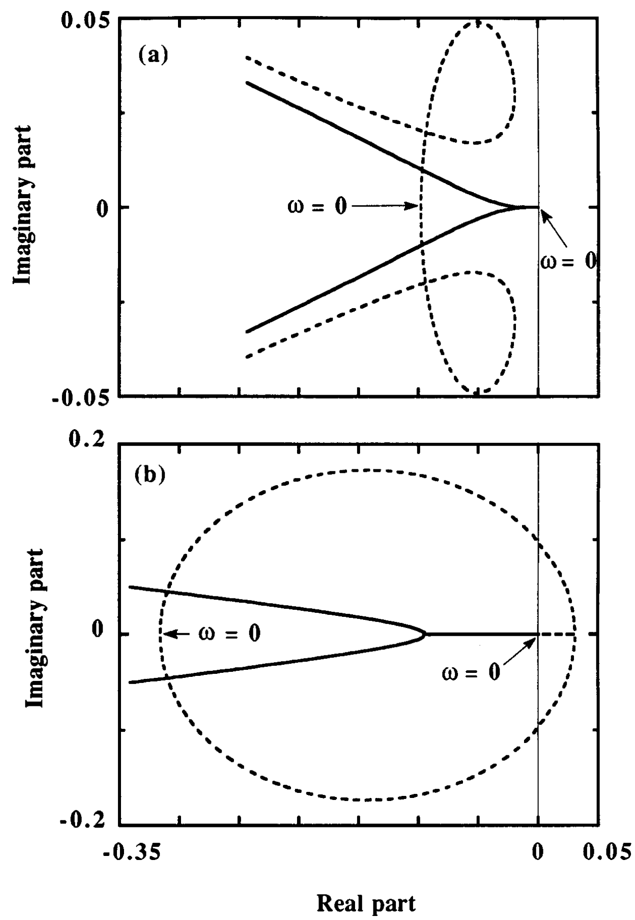

Fig. 1. Eigenvalue λ on the complex plane. The solid curve corresponds to the positive branch of Eq. (7), whereas the dashed curve corresponds to the negative branch. (a) Non-self-starting case. Parameter values are $P_{\mathrm{sat}} = 1$ mW, $B = 0.3$ ps^2, $D = 0.045$ ps^2, $\Gamma = 0.001$ W^{-1}, $K = 0.008$ W^{-1}, $T_0 = 1000$ ps, $g_0 = 3$, and $l = 0.05$. These parameters correspond to a dye laser with a weak saturable absorber. (b) Self-starting case. Parameter values are $P_{\mathrm{sat}} = 10$ mW, $B = 0.3$ ps^2, $D = 0.045$ ps^2, $\Gamma = 0.1$ W^{-1}, $K = 0.008$ W^{-1}, $T_0 = 10^8$ ps, $g_0 = 3$, and $l = 0.2$. These parameters correspond to a figure-eight laser.

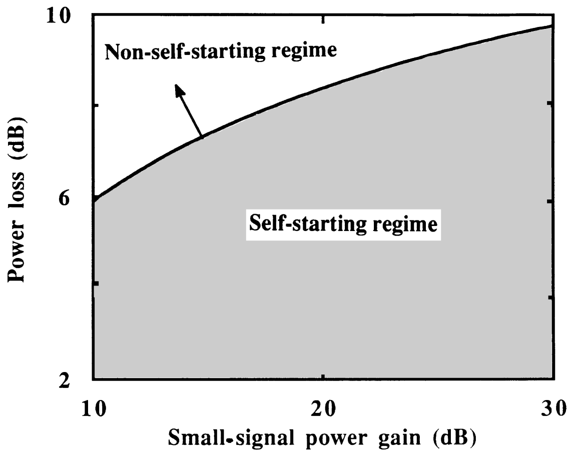

Fig. 2. Self-starting region in the $g_0 - l$ plane. The small-signal power gain is $\exp(2g_0)$. The parameter values are $T_0 = 10^6$ ps, $P_{\mathrm{sat}} = 1$ mW, $B = 0.3$ ps, $D = 0.045$ ps^2, $\Gamma = 0.001$ W^{-1}, and $K = 0.008$ W^{-1}.

section $\sigma \propto 1/T_0 P_{\mathrm{sat}}$ is predicted to aid self-starting. (3) A larger bandwidth also aids self-starting. Physically, that comes about because fluctuations exist over a wider range of frequencies. Mathematically, one finds from Eq. (7) that there is a slower movement of λ toward the left-hand plane with increasing ω^2 when B is small. (4) When D becomes large and positive, Eq. (1) becomes more analogous to the nonlinear Schrödinger equation, which is modulationally unstable, and self-starting is enhanced. Similarly, larger K values aid self-starting when D is positive.

The parameters g_0 and l influence self-starting through U_c in a complicated way. First we note that $U_c \epsilon_c/(1 + i\omega T_c) \sim g_0/i\omega T_0$ when $U_c^2 \gg P_{\mathrm{sat}}$. So when g_0 is fixed and l decreases, then U_c increases while $U_c \epsilon_c/(1 + i\omega T_c)$ is almost unchanged, and self-starting becomes easier. An increase in g_0 induces a linear increase in $U_c \epsilon_c/(1 + i\omega T_c)$; however, the increase in ΓU_c^2 is faster than linear, assuming that $U_c^2 \gg P_{\mathrm{sat}}$. Thus, increases in g_0 moderately aid self-starting.

In Fig. 2 we plot the region in the $g_0 - l$ plane in which self-starting can occur. The parameter set given in the figure caption is close to that of Fig. 1(b) and may again correspond to a figure-eight laser with external gain and a decorrelation time that is long enough for the laser to self-start. As expected, decreases in l and increases in g_0 enhance self-starting. We have shown in Ref. 7 that there exists one stable pulsed solution when the steady-state gain satisfies $g_s = g_0/(1 + P_{\mathrm{av}}/P_{\mathrm{sat}})$, where P_{av} is the average power. The shaded region that has an unstable cw mode has a stable pulsed solution and therefore is the region in which the laser will self-start.

In summary, we study the requirements for passively mode-locked lasers to self-start using the figure-eight laser as a practical example. We derive a dispersion relation $\lambda(\omega)$ that governs the stability of cw operation and use the criterion that $\mathrm{Re}(\lambda) > 0$ for some ω so that cw operation is unstable. Generally, increases in the relaxation time, bandwidth, dispersion, saturable absorption, Kerr nonlinearity, and the small-signal gain aid self-starting. Decreasing loss also aids self-starting.

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