Acoustic effect in passively mode-locked fiber ring lasers

A. N. Pilipetskii, E. A. Golovchenko, and C. R. Menyuk

Department of Electrical Engineering, University of Maryland, Baltimore, Maryland 21228-5398

Received October 20, 1994

We present a theory of acoustically induced pulse interaction in a ring fiber laser cavity. In most cases the acoustic interaction leads to pulse bunching, but in some cases it leads to regular pulse spacing. Our results compare well with the experimental data.

Passively mode-locked fiber ring lasers are an attractive source of picosecond and subpicosecond soliton-like pulses. In these lasers the basic dynamical balance that leads to passive mode locking of the individual pulses is due to a slow saturable gain provided by erbium-doped optical amplifiers, a fast-saturable absorption provided by nonlinear polarization rotation and polarization-selective elements, and a frequency filter that is provided by frequency-dependent cavity resonances combined with the saturable absorber.[1,2] These elements lead to no interpulse interactions, so that the laser cavity contains many pulses. One finds that they are free to move about with respect to one another, leading in most cases to a rich and complex dynamics.[3] However, recent experiments show that in some cases the pulses are all regularly spaced, indicating the presence of an interpulse interaction.[4,5] Not only is this result physically interesting but it has implications for potential applications that require regular pulse spacing.

Grudinin *et al.*[4] have proposed that the interpulse interaction could be due to acoustic waves stimulated by the pulses themselves. Because the lifetime of these waves in an optical fiber is of the order of 100 ns, this proposal is very plausible. In this Letter we examine this proposal both theoretically and computationally and show that under most circumstances the acoustic interaction leads to pulse bunching but under some circumstances it leads to regularly spaced pulses. Our results are consistent with the experiments. The acoustic interaction between passively mode-locked pulses in fiber ring lasers in analogous to the acoustic interaction between solitons in long-distance communication lines.[6,7]

Physically, the acoustic waves are created by electrostriction that is due to the large transverse electric field gradient in single-mode fibers. There are two groups of acoustic modes, torsional-radial TR_{2m} and radial R_{0m}, that can be excited by light pulses in single-mode fibers, but only the latter significantly perturb the fiber's refractive index $\delta n(t)$,[6] so only the latter must be taken into account. Each mode-locked pulse leaves in its train a perturbation $\delta n(t)$ that is due to the acoustic wave that lasts approximately 1 ns and repeats every 20 ns because of reflections from the fiber cladding up to more than 100 ns, so that mode-locked pulses affect those that come after them for more than 100 ns.

If one pulse follows the other at an interval T, then the first pulse changes the mean frequency of the other by[6]

$$\frac{\mathrm{d}\omega}{\mathrm{d}z} = -\frac{\omega}{c}\frac{\mathrm{d}(\delta n)}{\mathrm{d}t}\bigg|_{t=T}. \qquad (1)$$

Consequently this frequency shift results in the temporal shift of pulses relative to one another.

To investigate the acoustic interaction of pulses in the laser cavity, we assume that N pulses with equal amplitudes are circulating in the cavity, that the shape of these pulses is maintained by the laser itself, and that the pulses interact with one another only through the acoustic effect. These assumptions are plausible because the subpicosecond pulses produced by the fiber ring lasers that we are modeling[4] are too short to be shaped by the acoustic interaction, while there is no interpulse interaction as a result of the basic dynamics. We thus describe the sequence of pulses by merely using their temporal positions t_i and deviations from the central laser frequency $\delta\Omega_i$, and we find that

$$\frac{\mathrm{d}t_i}{\mathrm{d}z} = -\frac{(\delta\Omega_i)\lambda^2}{2\pi c}D, \qquad (2a)$$

$$\frac{\mathrm{d}(\delta\Omega_i)}{\mathrm{d}z} = -\frac{\omega}{c}\sum_l \frac{\mathrm{d}(\delta n)}{\mathrm{d}t}\bigg|_{t_i-t_l} - \beta\delta\Omega_i. \qquad (2b)$$

Here λ is the pulse wavelength, D is the average dispersion of the fiber ring cavity, and z is the propagation distance. The second term on the right-hand side of Eq. (2b) describes the frequency relaxation and comes from the frequency filtering in the modified Ginsburg–Landau equation governing the individual pulse shapes inside the laser cavity.[2] The sum in Eq. (2b) is taken over the acoustic response of all the preceding pulses. We also took into account that the acoustic wave radiated by the pulse itself influences the same pulse after one or more round trips in the laser cavity. In our simulations we assumed that there are no considerable changes in the mutual temporal distribution of pulses on the scale of the acoustic wave lifetime. Other parameters were $D = 15$ ps/(nm km), soliton duration $t(\mathrm{FWHM}) = 1$ ps, and the effective cross section of the fiber $S = 50\ \mu\mathrm{m}^2$, and the typical value of β in Eq. (2b) was estimated to be $\beta = 0.01\ \mathrm{m}^{-1}$. The evolution of pulse trains inside the cavity was calculated

0146-9592/95/080907-03$6.00/0

for different initial conditions, different numbers of pulses inside the cavity ($N = 5-300$) and different cavity lengths.

Our simulations indicate that the most probable evolution of an arbitrary pulse train in the laser cavity is that after some circulation time in the cavity the interaction force traps the pulses in bunches. A typical pulse bunch that can be generated is shown in Fig. 1. Note that the bunches themselves form a nearly periodic structure, that the period of this structure is approximately 200–500 MHz, and that each bunch is approximately 1.5 ns long and contains 3–10 pulses. The frequency range of the bunch sequence corresponds to the most efficient interaction of acoustic modes with light pulses.[8] Once the pulses are trapped in bunches, the subsequent evolution of the pulse train becomes very slow. In our calculations the typical time scale for bunch formation was 0.05–0.1 s. The generation of such bunches was experimentally observed in the research reported Refs. 3–5.

We now determine whether the acoustic interaction between the mode-locked pulses stabilizes or perturbs a periodic train of pulses inside the laser cavity. To do so, we consider a periodic train of N pulses circulating inside a ring laser cavity with period T and study the stability of such a train to the acoustic interaction as a function of the number of pulses N. A pulse in the sequence experiences a frequency shift

$$\frac{\mathrm{d}(\delta\Omega_i)}{\mathrm{d}z} = -\frac{\omega}{c}\sum_l \frac{\mathrm{d}(\delta n)}{\mathrm{d}t}\bigg|_{lT+\tau_i-\tau_l} - \beta\delta\Omega_i\,, \qquad (3)$$

where the sum is taken over the preceding pulses and τ_i is the deviation of the ith pulse from its position in the unperturbed periodic train. Recalling that there are N pulses in the cavity and that the ith pulse influences itself for several round trips in the cavity, we can approximate the expression for the frequency deviation by writing

$$\frac{\mathrm{d}(\delta\Omega_i)}{\mathrm{d}z} \approx -\frac{\omega}{c}\left[\sum_{l=1}^{\infty}\frac{\partial(\delta n)}{\partial t}\bigg|_{lT} + \tau_i\sum_{l=1}^{\infty}\frac{\partial^2(\delta n)}{\partial t^2}\bigg|_{lT} - \tau_i\sum_{l=1}^{\infty}\frac{\partial^2(\delta n)}{\partial t^2}\bigg|_{lNT} - \sum_{l=0}^{\infty}\sum_{k=1}^{N-1}\tau_k\frac{\partial^2(\delta n)}{\partial t^2}\bigg|_{kT+lNT}\right] - \beta\delta\Omega_i\,. \qquad (4)$$

The first sum inside the brackets in the right-hand part of relation (4) can be neglected because it leads to an equal change of frequency for all the pulses and consequently does not affect their mutual positions. Combining relation (4) and Eqs. (2), we obtain a second-order linear system of equations for the pulse deviation from the unperturbed position. To solve this system of equations for τ_i we can represent τ_k as a sum of harmonics:

$$\tau_k(z) = \sum_{m=1}^{N-1}\tau^{(m)}(z)\exp[2\pi j(k-i)m/N]\,, \qquad (5)$$

where k is the pulse number and m is the harmonic number. We then obtain linear equations for each $\tau^{(m)}(z)$. We look for a solution in the form $\tau^{(m)}(z) \sim \exp(\lambda_m z)$ and determine the eigenvalues λ_m. The stability requirement is that $\mathrm{Re}(\lambda_m) < 0$ for all harmonics. The train of pulses will be unstable even if $\mathrm{Re}(\lambda_m) > 0$ for only one harmonic. A typical result obtained from this analysis is shown in Fig. 2 for a 60-m-long cavity. The dots give the number of pulses N inside the cavity for which the periodic pulse train is stable. For the other values of N the pulse train is unstable. Figure 2 is consistent with the experimentally obtained data of Refs. 4 and 5. We checked the results of the linear stability analysis by direct numerical calculations by using Eqs. (2).

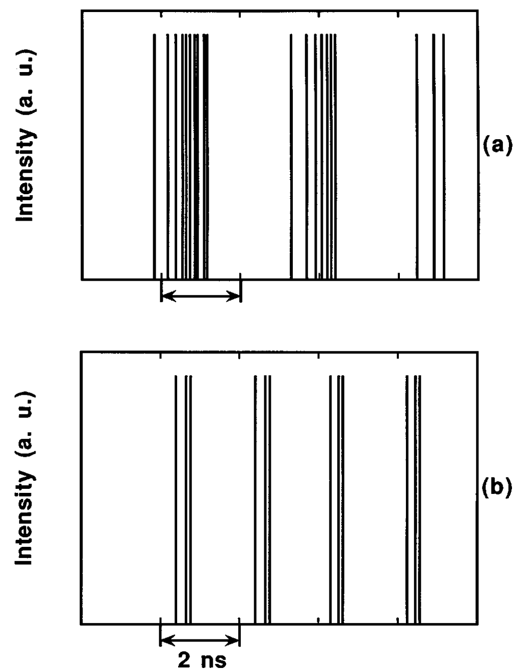

Fig. 1. (a) $N = 20$ and (b) $N = 12$ mode-locked pulses trapped into bunches by the acoustic interaction. Note that picosecond pulses on the nanosecond time scale look like delta functions.

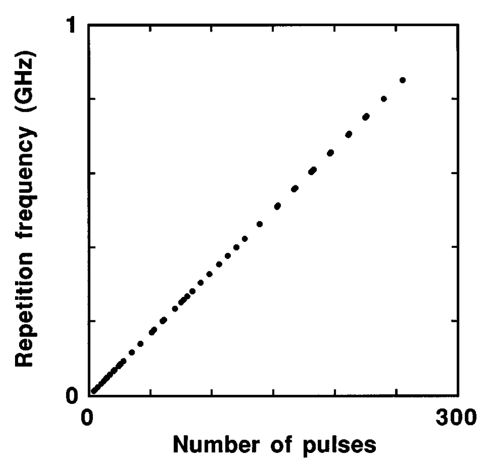

Fig. 2. Frequency dependence as a function of N of stable harmonically mode-locked operation. Dots are shown only at N values at which the operation is stable, and regular pulse spacing is observed.

When N has an unstable value, the long-range interaction destroys the initial periodic sequence of pulses and eventually traps them in bunches. When N has a stable value, the pulse trains remain regularly spaced so that there are certain values of N for which the generation of a regularly spaced pulse sequence is possible. It should be noted that the exact values of N for which the periodic pulse train is stable depend on the cavity length. The corresponding frequencies also depend on the cavity length and lie around the eigenfrequencies or the subharmonics of eigenfrequencies of the acoustic modes.

In this Letter we have shown that the acoustically induced long-range interaction in a ring fiber laser cavity can either trap pulses in bunches or lead to regularly spaced pulse sequences. The results of our theory coincide with the experimental data. We should emphasize that the acoustic interaction reveals itself only in passively mode-locked lasers and in most cases is undesirable because it traps pulses in bunches. On the other hand, the perturbation of the fiber effective refractive index that is due to the acoustic vibrations is rather small (10^{-11}–10^{-12}), so the long-range acoustic interaction of light pulses can be easily eliminated by active mode locking of pulse trains, by the insertion of subring cavities, or by the use of internal Fabry–Perot stabilizers.

This research was supported by the Advanced Research Projects Agency through the U.S. Air Force Office of Scientific Research and by the National Science Foundation.

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