Acoustic effect and correlated errors in soliton information transmission

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The acoustic effect significantly increases the timing jitter of solitons in communication lines. We calculate the correlation in near-neighbor soliton time shifts that result from the acoustic interaction and show that the acoustic effect can cause correlated errors that cannot be corrected with standard, simple error-correction codes such as the Hamming code. © 1996 Optical Society of America

Solitons that propagate in an optical fiber experience timing jitter that limits both the bit rate and the information transmission distance. This timing jitter is caused by several effects: the Gordon–Haus effect, the polarization effect, and the acoustic effect.[1,2] At bit rates in excess of 10 Gbits/s, the acoustic effect becomes the dominant cause of the timing jitter at distances greater that a few thousand kilometers.[1,2] The acoustic effect is created by the large transverse gradient of the electric fields in the optical fiber that results from the soliton pulses. These large field gradients electrostrictively excite acoustic waves that influence later solitons. The acoustic wave perturbs the effective refractive index of the fiber, leading to changes in the frequencies and temporal locations of the solitons. The excited acoustic wave propagates transversely to the fiber axis; so the perturbation of the effective refractive index of the fiber changes on a time scale of 1 ns—the time that it takes for the acoustic wave to cross the fiber core area. We thus expect that neighboring solitons in a high-bit-rate transmission system, operating at more than 5 Gbits/s, will experience correlated time shifts. Obviously the correlated time shifts of solitons can cause correlated errors in information transmission. Our goal in this Letter is to calculate the correlation between time shifts of solitons and to discuss the consequences for information transmission.

The physical source of the Gordon–Haus and polarization effects is spontaneous emission in the erbium-doped fiber amplifiers. From the standpoint of communication theory, both these effects are sources of additive white noise.[3] Although chromatic dispersion, polarization-mode dispersion, and in-line filtering in combination with the channel nonlinearity seriously complicate the calculation of the bit error rate because of these noise sources,[4] these noise sources lead to only a very weak intersymbol interference in a soliton system and to nearly uncorrelated errors from bit to bit. By contrast, the acoustic effect is not a noise source at all but rather a source of intersymbol interference.[3] The effect of the acoustic effect on any particular bit depends in a completely deterministic way on the bits that preceded it, and errors are highly correlated from bit to bit.

To date, there has been little or no thought given to the possible effect of methods such as error-control coding or feedback (equalization) in eliminating errors that are due to the optical fiber transmission line in soliton communications. Given the power of these techniques, it is apparent that more thought must be given to their potential. In this Letter we open the discussion by pointing out the inadequacy of a simple Hamming code in dealing with the highly correlated errors that are due to the acoustic effect.

Each soliton that propagates in a communication line excites an acoustic wave and perturbs the fiber refractive index, and this perturbed refractive index $\delta n(t)$ affects the subsequent solitons.[2] If one pulse follows another at an interval T, then the first pulse changes the mean frequency of the other by

$$\frac{\mathrm{d}\omega}{\mathrm{d}z} = -\frac{\omega}{c}\,\frac{\mathrm{d}(\delta n)}{\mathrm{d}t}\bigg|_{t=T} . \tag{1}$$

The index perturbation $\delta n(t)$ is proportional to the energy of the first pulse, and its functional form may be found in Ref. 2. This frequency shift leads to a temporal shift of the pulses relative to each other. A data stream that consists of an arbitrary sequence of 1's and 0's is physically represented in a soliton communication line by a sequence of solitons that are separated from one another by an interval T and appear with a probability equal to 1/2. The ith soliton in the pulse sequence is then described by the temporal position t_i and a deviation from the central signal frequency $\delta\Omega_i$:

$$\frac{\mathrm{d}t_i}{\mathrm{d}z} = -\frac{(\delta\Omega_i)\lambda^2}{2\pi c}\,D\,, \tag{2a}$$

$$\frac{\mathrm{d}(\delta\Omega_i)}{\mathrm{d}z} = -\frac{\omega}{c}\sum_l \frac{\mathrm{d}(\delta n)\lambda^2}{\mathrm{d}t}\bigg|_{t_i-t_l}\,, \tag{2b}$$

where λ is the soliton wavelength, D is the average dispersion in the communication line, and z is the propagation distance. The summation in Eq. (2b) is taken over the acoustic responses of all the preceding pulses.

We wish to calculate the correlation function between time shifts of the pulses:

$$f(N) = \frac{\langle \tau_i \tau_{i+N}\rangle - \langle \tau_i\rangle\langle \tau_{i+N}\rangle}{\sigma^2}\,, \tag{3}$$

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where $\sigma \equiv \langle \tau_i^2 \rangle - \langle \tau_i \rangle^2$, $\tau_i = t_i(z) - t_i(0)$ is the time shift of the pulse in the time slot with number i from its initial position and σ is the variance of the acoustically induced timing jitter. We assume that the process distribution for the 1 bits and the 0 bits is stationary so that f does not depend on i. Taking into account that the probability of having a soliton in each bit is equal to 1/2 and that soliton time shifts are much smaller than 1 ns, one finds that

$$f(N) = \frac{\sum_{l=1}^{\infty} \frac{d\delta n}{dt}\Big|_{lT} \frac{d\delta n}{dt}\Big|_{(l+N)T}}{\sum_{l=1}^{\infty} \left(\frac{d\delta n}{dt}\Big|_{lT} \right)^2}, \quad (4)$$

where T is the bit period. The correlation function of the soliton time shifts at a bit rate of 20 Gbits/s is presented in Fig. 1. The long-term correlations shown in Fig. 1(a), on a time scale of approximately 20 ns, are due to reflections from the fiber cladding boundary, whereas short-term correlations [Fig. 1(b)], on a time scale of 2 ns, are due to the finite transit time of the acoustic waves through the core. In an unfiltered soliton system the acoustic effect leads to the timing jitter of pulses with variance

$$\sigma = 4.8 \frac{D^2 F^{1/2}}{\tau} z^2, \quad (5)$$

where D is the fiber dispersion in units of ps/(nm km), z is the propagation distance in units of Mm, and F is the bit rate in gigabits per second.[1,2] With guiding filters the timing jitter is reduced by a factor $2/\beta z$, where β is the frequency damping coefficient,[1,2] but, in any case, the time shifts of neighboring solitons are strongly correlated (Fig. 2). The correlation between time shifts of two neighboring solitons for a bit rate of more than 10 Gbits/s can be approximately expressed as

$$f \approx 1 - \frac{1.4}{F}, \quad (6)$$

where F is the bit rate in gigabits per second. The timing jitter that accumulates along the transmission line can cause errors when a soliton leaves its time slot.

We note that the probability that a soliton leaves its time slot is not identical to the probability of error. First, it is possible that a soliton will move into a slot where there was a 0, in which case this movement causes two errors; it is also possible for a whole chain of solitons to move into neighboring time slots, leading to two errors for the whole train. Second, a soliton is counted only when it is in the middle 50–80% of the time slot, depending on the detection scheme. Thus we focus on the probability of solitons' leaving their time slots rather than the probability of error, although the two are closely related.

In the case of acoustically induced timing jitter, one finds that when one soliton moves out of its time slot the soliton in the neighboring time slot will also have a large enough time shift to move out of its time slot with high probability, leading to correlated errors. This correlation can have devastating consequences for simple error-correction codes that assume that errors are independent. We illustrate this possibility with an example by using a simple Hamming code.[3] In this code, k parity bits are added to an n-bit word, resulting in a new word with $n + k$ bits. The positions in the new word that correspond to powers of 2 are assigned the parity bits. The remaining positions are assigned the data bits. We simulated the propagation of a data stream with 10,000 words through an optical fiber, calculating the changes in the pulse frequencies and pulse positions with Eqs. (2). The initial data stream consisted of 8-bit words following one another with arbitrarily chosen 0's and 1's. To each 8-bit word we added 4 parity bits, so that the data stream that we simulated was represented by a sequence of 12-bit words. Our simulation parameters were $D = 0.2$ ps/(nm km), $\tau_{sol} = 10$ ps, $F = 20$ Gbits/s, and

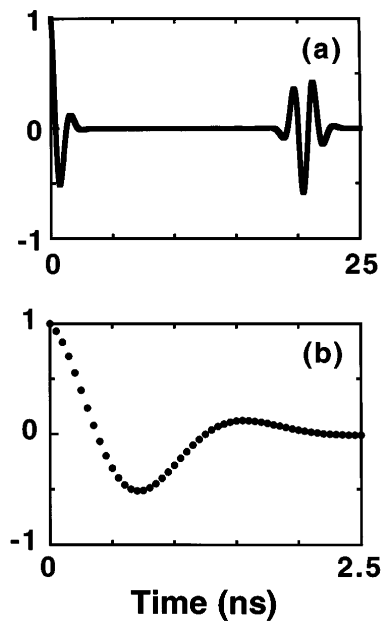

Fig. 1. Correlation function for the acoustically induced timing jitter at a bit rate of 20 Gbits/s on two different time scales: (a) 0–25 ns, (b) 0–2.5 ns. Dots indicate the pulse positions.

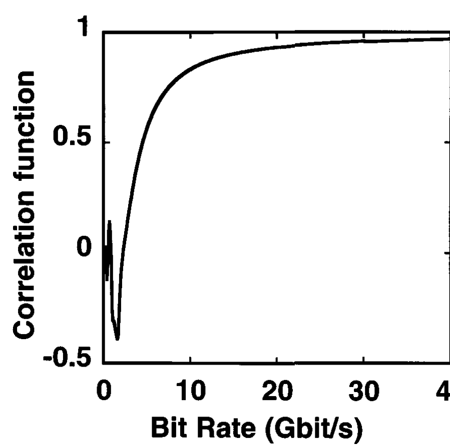

Fig. 2. Correlation between time shifts of neighboring solitons depending on the bit rate.

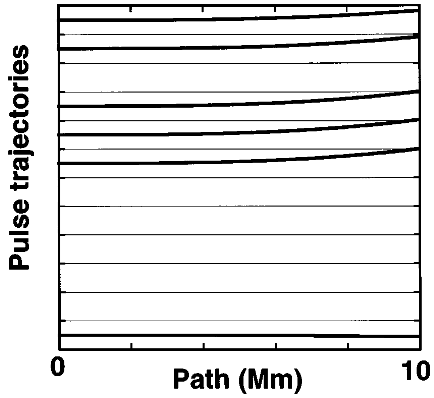

Fig. 3. Evolution of the soliton maxima in one 12-bit word in our simulated bit stream. Our parameters are bit rate $F = 20$ Gbits/s, dispersion $D = 0.25$ ps/(nm km), and soliton width $\tau = 10$ ps. The trajectories of the pulse maxima are shown by thick solid curves, and the grid indicates the boundaries of the time slots.

$z = 10$ Mm. For these parameters the probability of a single soliton's moving out of its time slot approximately equaled 10^{-2}, which would be unacceptably high in a real communication system but allowed us to easily study the correlated time shifts. For the actual string that we studied, we found that the actual probability that a soliton leaves its time slot is 0.96×10^{-2}, while the actual probability of soliton's leaving its time slot given that the preceding soliton leaves its time slot is 0.44. These results are consistent with the theoretically expected values.

A typical example of a word in which an error occurred is shown in Fig. 3. Before transmission, this word was an 8-bit word, 00011011. To use the Hamming code we assigned the four parity bits, P_1 = XOR of bits (3, 5, 7, 9, 11) = 1, P_2 = XOR of bits (3, 6, 7, 10, 11) = 0, P_4 = XOR of bits (5, 6, 7, 12) = 0, and P_8 = XOR of bits (9, 10, 11, 12) = 1, to the first, second, fourth, and eighth places. The XOR operation equals 1 when there is an odd number of 1's in the variables and equals 0 when there is an even number of 1's. We thus obtained a 12-bit word 100000111011. When the 12 bits are received after passing through the transmission line they are checked for the errors as follows: C_1 = XOR of bits (1, 3, 5, 7, 9, 11), C_2 = XOR of bits (2, 3, 6, 7, 10, 11), C_4 = XOR of bits (4, 5, 6, 7, 12), and C_8 = XOR of bits (8, 9, 10, 11, 12). For a single error the binary result $C = C_8C_4C_2C_1 \neq 0$ will indicate the error and the bit position where the error occurs. In soliton transmission, a 1 is represented by a soliton and a 0 is represented by the absence of a soliton. Figure 3 shows that after the propagation distance $z = 10{,}000$ km there are already three solitons that have moved out of their time slots, and the 12-bit word is represented by the sequence 100000011111. When this word is received after the transmission line, the bits are checked for errors as follows: C_1 = XOR of bits (1, 3, 5, 7, 9, 11), C_2 = XOR of bits (2, 3, 6, 7, 10, 11), C_4 = XOR of bits (4, 5, 6, 7, 12), and C_8 = XOR of bits (8, 9, 10, 11, 12). With a single error, the binary result $C = C_8C_4C_2C_1 \neq 0$ indicates the error and the bit position where the error is located. Here $C = 1101$ equals 13, which incorrectly indicates the wrong bit. We conclude from this example that simple error-correction codes that work well when bits are subject to independent errors can fail in the case of the acoustically induced timing jitter.

In this Letter we showed that errors that are due to the acoustic effect are highly correlated and that this correlation will have a devastating effect on simple error-correction codes, such as a simple Hamming code, when the acoustic effect dominates, which occurs at data rates above 10 Gbits/s. By contrast, errors that are due to spontaneous emission noise are nearly independent, and we therefore expect a simple Hamming code to work well at data rates below 10 Gbits/s.

A variety of techniques exist that could be of potential use in reducing errors that result from to the acoustic effect. Since the acoustic effect is a source of intersymbol interference, feedback (equalization) techniques are promising.[3] Some error-control coding techniques that deal with bursts are promising as well.[3] We believe that we have only scratched the surface of this important area of investigation.

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