Minimum channel spacing in filtered soliton wavelength-division-multiplexing transmission

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We study a soliton transmission line with sliding-frequency filters, and we determine the limits on soliton stability imposed by the value of the free spectral range of Fabry–Perot étalon filters. From these limits, we infer the minimum channel spacing that is possible in a soliton wavelength-division-multiplexing system.

One of the advantages of soliton transmission lines is the natural way in which they can be combined with wavelength-division multiplexing (WDM). The total data rate is determined by the individual channel data rate and the number of channels. The largest number of channels that can be used is determined in turn by the minimum and maximum frequency spacing of channels. In a soliton transmission line without filtering, the minimum channel spacing for WDM is determined by timing jitter induced by soliton collisions[1] or the frequency shift that occurs when solitons collide in the detector[2]; both effects limit the spacing to a minimum of approximately four soliton spectral widths.[2] The maximum spacing is determined by the requirements that soliton collisions occur over several amplifiers[1] and that the channel gain differential be nearly zero[2]; this limitation is typically several nanometers. In a soliton transmission line with Fabry–Perot étalons as sliding-frequency guiding filters, the channel spacing is determined by the separation between the frequencies at the peaks of the filter transmission function. By changing the free spectral range of the filter one may bring the WDM channels closer. On the other hand, the filter characteristics cannot be changed arbitrarily because certain restrictions are imposed on the filter by the requirement that the solitons remain stable during propagation.[3,4] In this Letter we analyze how closely one can bring WDM channels with Fabry–Perot étalons while maintaining stable soliton propagation.

The propagation equation in soliton units is[3,5]

$$\frac{\partial u}{\partial z} - i\omega_f'\tau u - i\,\frac{1}{2}\,\frac{\partial^2 u}{\partial \tau^2} - i|u|^2 u = \frac{\alpha}{2} + \int_0^\infty f(\theta)u(\tau-\theta, z)\mathrm{d}\theta\,, \quad (1)$$

where α is the amplifier excess gain, ω_f' is the constant sliding rate, and $f(\tau)$ is the response function of a distributed Fabry–Perot étalon filter. The Fourier transform $\tilde{f}(\omega)$ of $f(\tau)$ may be written as

$$\tilde{f}(\omega) = \frac{z_d}{l_f}\ \ln\ \frac{1-R}{1-R\ \exp[i(\omega-\omega_f)2d/c\tau_0]}\,, \quad (2)$$

where c is the speed of light, $z_d = \tau_0^2 2\pi c/D\lambda^2$ is the dispersion length of a soliton with duration $\tau_0 \equiv t_0/1.76$ (with t_0 the pulse FWHM), λ is the wavelength, D is the dispersion coefficient, l_f is the filter separation along the transmission line, d is the filter air gap, and R is the reflection coefficient. We recall that the free spectral range equals $c/2d$, and we note that Eq. (1) is written in retarded coordinates that accelerate with the soliton. In this set of coordinates the filter is stationary, whereas the soliton mean frequency is shifting at the filter sliding rate ω_f'.[5]

In most previous analyses the filter function was expanded in a Taylor series and only the second- and the third-order terms of this expansion were included. Through second order, this expansion term approximates the filter shape by a parabola with the curvature η_2 defined as

$$\eta_2 = \frac{1}{2}\,\frac{R}{(1-R)^2}\,\frac{8\pi}{Dl_f c}\left(\frac{d}{\lambda}\right)^2. \quad (3)$$

In this approximation it was recently shown that soliton propagation remains stable up to $\eta_2 = 0.4$,[4] and the soliton mean frequency ω_0 at equilibrium is offset from the filter frequency ω_f by

$$\Delta\omega = \omega_0 - \omega_f = -3\omega_f'/4\eta_2 A^2\,, \quad (4)$$

where A is the soliton amplitude. The relation among the excess gain α, soliton amplitude A, and the filter curvature η_2,

$$\alpha = \frac{2\eta_2}{3A^2} + 2\eta_2(\Delta\omega)^2\,, \quad (5)$$

provides the condition for stable soliton propagation where gain and losses are balanced, and the condition $\omega_f' < \omega_{\mathrm{cr}}' = 2(2/27)^{1/2}\eta_2$ is imposed by the filter strength on the sliding rate.[3] The next approximation, which takes into account the third-order filter term, results in correction to the expression for $\Delta\omega$ given by Eq. (4) and implies that upsliding in frequency is better than downsliding for practical soliton transmission.[6]

With the decrease of the filter transmission channel separation the changes in $\tilde{f}(\omega)$ on the scale of the soliton spectral width $\Delta\omega_0$ become too rapid to be usefully approximated by the first few terms of a Taylor expansion. We therefore study the impact of the full filter function $\tilde{f}(\omega)$ given by Eq. (2) on soliton

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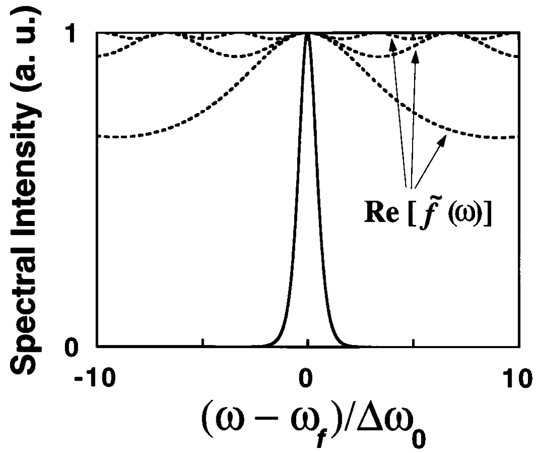

Fig. 1. Soliton spectrum (solid curve) and the real part of the Fabry–Perot étalon filter Re[$\tilde{f}(\omega)$] (dashed curves) for different free spectral ranges (FSR's): FSR/$\Delta\omega_0$ = 16, 7, and 3.5.

propagation. The shape of $\tilde{f}(\omega)$ at several different values of the free spectral range together with the soliton spectrum is shown in Fig. 1. We simulated soliton propagation governed by Eq. (1) for the input pulse $u_0 = \text{sech}(\tau)$ with the initial mean frequency ω_0 centered under the peak filter transmission frequency ω_f. This initial pulse does not correspond to the equilibrium soliton state in the transmission line, and it therefore sheds radiation, which is damped, while evolving toward the equilibrium state, when it exists, resulting in a pulse with some amplitude A, duration τ, and mean frequency offset $\Delta\omega$ from ω_f. When no equilibrium exists, the calculations indicate that the initial pulse is unstable. Note that the numerical observation of instability is not enough to ensure that there is no equilibrium because the initial conditions could be outside the basin of attraction. In practice, however, as we discuss below, there is a physical reason to believe that no equilibrium exists when solitons are unstable in our calculations. In our calculations we set the sliding rate to be $\omega_f' = 0.8\omega_{\text{cr}}'$, and we used upsliding. The other parameters that we used in our calculations were pulse duration $t_0(\text{FWHM}) = 1.76\tau_0 = 20$ ps, filter separation along the transmission line $l_f = 30$ km, and fiber dispersion coefficient $D = 0.5$ ps/(nm km). We also used the value of excess gain given by Eq. (5), so that in all cases in which we found stable solitons their duration and amplitude nearly equaled those of the initial pulse.

In Fig. 2 we show the evolution of the peak pulse amplitude A and the mean frequency offset $\Delta\omega$, setting the filter curvature $\eta_2 = 0.1$ and using different values of the filter free spectral range. We first note that, as the free spectral range decreases, the value of $\Delta\omega$ grows. This effect is predicted by the full filter function $\tilde{f}(\omega)$ and is not contained in the third-order Taylor expansion. When the filter peaks are closer than four spectral widths of the soliton, $\Delta\omega$ becomes too large for the chosen value of excess gain to balance the filter loss, and the pulse dies after some distance of propagation, as shown in Fig. 2. From this physical argument, we conclude that there is no longer a stable soliton solution.

From a practical standpoint, large $\Delta\omega$ is not desirable for soliton information transmission. When $\Delta\omega$ is large, the filter curvature at the soliton mean

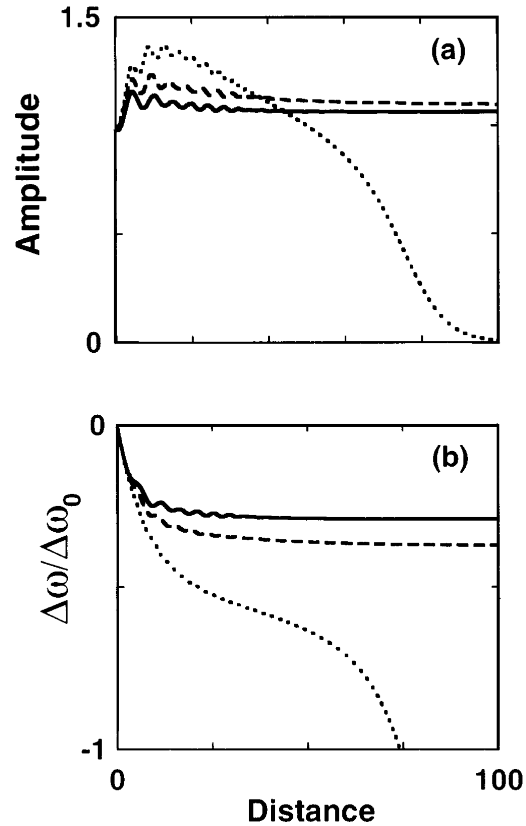

Fig. 2. (a) Soliton peak amplitude A and (b) mean frequency offset from the filter frequency $\Delta\omega$ as a function of distance z given in soliton periods z_0, obtained through numerical solution of Eq. (2) for filters with curvature $\eta_2 = 0.1$ and different free spectral ranges (FSR's): FSR/$\Delta\omega_0$ = 10 (solid curves), FSR/$\Delta\omega_0$ = 6 (dashed curves), and FSR/$\Delta\omega_0$ = 3.5 (dotted curves).

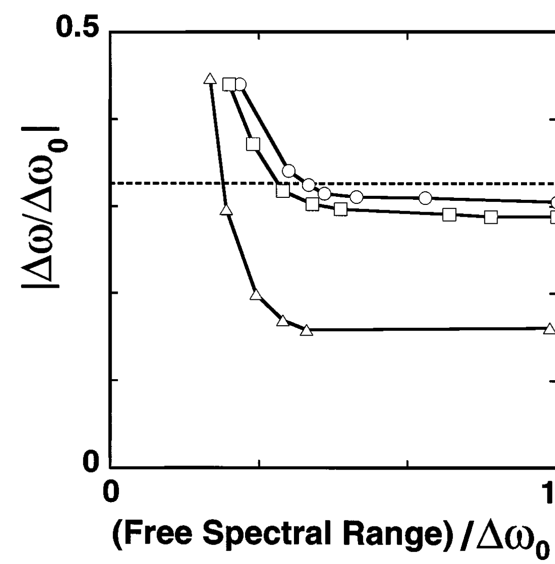

Fig. 3. Absolute value of the soliton mean frequency offset from the filter peak frequency at equilibrium $\Delta\omega$ as a function of the filter free spectral range for the filter curvatures $\eta_2 = 0.03$ (circles), $\eta_2 = 0.1$ (squares), and $\eta_2 = 0.3$ (triangles).

frequency is smaller than in Eq. (3), indicating that more gain is required for stable soliton propagation. Our calculations showed that if we increase the value of excess gain by 20% from the value given by Eq. (5) stable soliton propagation may be obtained even when $\Delta\omega = 0.6$. However, the soliton spectral width at equilibrium is considerably broader than the initial spectral width because the ratio between gain and loss that yields the resultant pulse width has changed. An increase of the excess gain in a real communication line implies an the increase of the soliton timing jitter, and the increase of the spectral width implies WDM channel cross talk. We thus limit our considerations to the excess gain α given by Eq. (5), and we take the free spectral range at which the filter fails to guide the soliton as the limiting case for WDM channel spacing.

We have discussed a relatively weak filter with $\eta_2 = 0.1$. As η_2 increases, the pulse shape at equilibrium becomes distorted, developing sidebands in the time domain. Moreover, its spectrum becomes oscillatory,[3] imposing another limitation on the WDM channel separation. We have investigated the pulse evolution over 100 soliton periods for different filter strengths η_2. In Fig. 3 we show $\Delta\omega$ as a function of the filter's spectral range. Each marked point in Fig. 3 corresponds to a computational result. The dashed line shows the portion of $\Delta\omega$ imposed on the soliton by sliding alone, which is given by Eq. (4). We first observe, as expected from the results of Ref. 6 with upsliding, that $\Delta\omega$ decreases as η_2 increases. Also, when the free spectral range is large, we find that $\Delta\omega$ is smaller than predicted by Eq. (4). The first marked point on each of the curves is the soliton stability limit, corresponding to the disappearance of the pulse at $\eta_2 = 0.1$ and pulse shape distortion at $\eta_2 = 0.3$. It is remarkable that the limiting value of $\Delta\omega$ is nearly the same for different filter strengths η_2 and lies near 0.45 of the soliton spectral FWHM, $\Delta\omega_0$. From Fig. 3 we infer that the optimal value of the filter peak separation is approximately 5 FWHM of the soliton spectrum because under this condition soliton propagation is stable for almost any filter strengths of interest.

In conclusion, we have numerically investigated the limitation that Fabry–Perot filters impose on the frequency spacing of WDM channels. We found that the minimal value of filter peak frequency separation is 5 FWHM of the soliton spectrum. This value is only slightly larger than in an unfiltered system and indicates that one need only pay a minor penalty in spacing of WDM channels when using sliding filters.

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