Dispersion-managed soliton dynamics

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We obtain and solve algebraic eigenvalue equations that predict the dependence of the pulse energy of a dispersion-managed soliton on pulse duration, chirp, and dispersion-map parameters. We demonstrate that a variational ansatz based on a Gaussian pulse shape remains useful even when the actual pulse shape is not Gaussian, and we show that the enhancement factor saturates as the pulse duration decreases.

Impressive results obtained recently indicate that dispersion-managed solitons have advantages relative to standard solitons in either constant-dispersion or even dispersion-decreasing optical fibers. Single-channel distance × rate products have been achieved that exceed the best results for standard solitons without soliton control obtained with either sliding filters or active retiming.[1,2] Theory indicates that there is a reduced Gordon–Haus jitter at any given dispersion,[2–4] a reduction in the mutual interaction,[5] and either comparable or reduced timing jitter in a wavelength-division-multiplexed system.[6] Numerical simulations indicate that as the difference between the dispersion in the spans of the dispersion map increases, with the average dispersion kept fixed, the pulse shape at its point of maximum compression changes from a hyperbolic secant shape to a Gaussian shape[1,2] to a shape that is sinclike.[5]

A key theoretical distinction between solitons and nonreturn-to-zero signals is that the former can be analyzed with reduced models that follow only a few of the soliton's parameters, whereas the latter with few exceptions must be analyzed with the nonlinear Schrödinger equation and its extensions. This distinction is conceptually important,[7] but it also has practical implications because it implies that the soliton systems are simpler to analyze and design. In particular, theoretical predictions of soliton amplitude and timing jitter based on reduced models have yielded impressive agreement; there is no analogous result for nonreturn-to-zero systems.

It is natural to ask whether it is possible to obtain a reduced description of the dispersion-managed solitons that can accurately reproduce the behavior during an entire period of the evolution. Here we answer this question affirmatively. Aside from this conceptual issue, we had an important practical motivation for developing a reduced model. Determining the shape of the periodically stationary pulse as a function of pulse duration and system parameters directly from a numerical solution of the nonlinear Schrödinger equations is computationally time consuming, requiring considerable trial and error[5]—particularly if the launching point is a point of maximum expansion where the pulse is chirped. Using our reduced approach, we have been able to explore a wider range of the parameter space than was possible previously and to show that the enhancement factor ultimately saturates as the pulse duration decreases, a result that was later verified computationally.

Our reduced model is derived by use of Anderson's variational approach[8] with a Gaussian ansatz. The Gaussian ansatz is used both because of its simplicity and because computational results indicate that pulses have this shape at certain values of the dispersion difference. We show, however, that it yields reasonably accurate results even when the shape is not Gaussian. Our starting point is the nonlinear Schrödinger equation, modified to include a spatially varying dispersion $D(z)$ and gain $g(z)$:

$$i\,\frac{\partial q}{\partial z} + \frac{1}{2}\,D(z)\,\frac{\partial^2 q}{\partial t^2} + |q|^2 q = ig(z)q\,, \tag{1}$$

where q is the normalized field amplitude and z and t are dimensionless distance and time, respectively. We use the ansatz

$$q = A\exp\left[\left(-\frac{1}{\tau^2} + i\alpha\right)t^2 + i\sigma\right], \tag{2}$$

where, physically, A, τ, α, and σ indicate the pulse's amplitude, duration, chirp, and phase, respectively. The variational method yields the following equations:

$$\begin{aligned}\frac{\mathrm{d}\tau}{\mathrm{d}z} &= 2D\alpha\tau\,,\\ \frac{\mathrm{d}\alpha}{\mathrm{d}z} &= 2D\left(\frac{1}{\tau^4} - \alpha^2\right) - \frac{S}{\tau^3}\,,\end{aligned} \tag{3}$$

where $S = (U_0/\sqrt{2})G(z)$, $G = \exp[2\int_0^z g(z)\mathrm{d}z]$, and $A^2\tau = U_0$ is constant as a function of z. It follows from Eqs. (3) below that the equation for the pulse duration in each homogeneous span is

$$\frac{\mathrm{d}^2\tau}{\mathrm{d}z^2} - \frac{4D^2}{\tau^3} + \frac{2DS}{\tau^2} = 0\,. \tag{4}$$

Equations similar to Eqs. (3) were previously obtained by Gabitov *et al.*[9] and by Matsumoto and Haus,[10] who verified numerically that the equations work well with a Gaussian ansatz in the parameter region in which numerical results show that the pulse shape is Gaussian. Here we demonstrate that the equations are useful even when the pulse shapes are non-Gaussian. We also note that Yang and Kath[11] used

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another reduced approach, based on perturbation theory, to derive the numerically observed[4,5] increase in the enhancement factor when the pulse duration decreases. Here we show that the enhancement factor saturates as the pulse duration decreases.

From here on, we focus on a lossless medium to relate our results to those that have been published,[4,5] although we stress that our formulation applies equally well when loss is compensated for by gain. In the case considered here S is constant as $G(z) = 1$. The solution to Eq. (4) in a single span can be written implicitly as

$$(C\tau^2 + 2f\tau - 1)^{1/2} - \frac{f}{\sqrt{C}} \ln\{C\tau + f + [C(C\tau^2 + 2f\tau - 1)]^{1/2}\} = 2CDz\,, \quad (5)$$

where $C = \alpha_0{}^2\tau_0{}^2 + 1/\tau_0{}^2 - 2f/\tau_0$, $f = S/2D$, and α_0 and τ_0 are the chirp and the pulse durations, respectively, at the beginning of the span, as shown in Fig. 1. Equation (4), which is symmetric with respect to z, has a solution in which $\tau(z)$ is strictly symmetric about the midpoint of each span of the dispersion map and for which $\alpha(z)$ is strictly antisymmetric. We search for periodically stationary solutions $\tau(z)$ and $\alpha(z)$ with the period of the dispersion map length. The symmetry condition together with the periodicity condition implies that the solutions satisfy $\tau(L_1) = \tau(L_1 + L_2) = \tau_0$ and $\alpha(L_1 + L_2) = -\alpha(L_1) = \alpha_0$, where L_1 and L_2 correspond to the lengths of the spans with dispersions D_1 and D_2, respectively. Combining these conditions with Eq. (5), we obtain the following algebraic eigenvalue equations for determining the boundary values of α_0 and τ_0:

$$\frac{f_1}{\sqrt{C_1}} \ln\left(\frac{C_1\tau_0 + f_1 + \sqrt{C_1}\,\alpha_0\tau_0{}^2}{\sqrt{f_1{}^2 + C_1}}\right) = C_1D_1L_1 + \alpha_0\tau_0{}^2,$$

$$\frac{f_2}{\sqrt{C_2}} \ln\left(\frac{C_2\tau_0 + f_2 - \sqrt{C_2}\,\alpha_0\tau_0{}^2}{\sqrt{f_2{}^2 + C_2}}\right) = C_2D_2L_2 - \alpha_0\tau_0{}^2, \quad (6)$$

where $f_{1,2} = f(D_{1,2})$ and $C_{1,2} = C(D_{1,2})$. The pulse durations at the midpoints of the spans with dispersion D_1 and D_2 are then defined as

$$\tau_{1,2} = \frac{1}{C_{1,2}}\left(-f_{1,2} + \sqrt{f_{1,2}{}^2 + C_{1,2}}\right)\cdot \quad (7)$$

To validate the variational approach we first compare the pulse dynamics given by Eqs. (5) and (6) with a complete numerical solution of Eq. (1), using the split-step method. The first case, in Fig. 2(a), corresponds to $\gamma = 2[(D_1 - \overline{D})L_1 - (D_2 - \overline{D})L_2]/t_{\mathrm{FWHM}}{}^2 = 2.89$, where $\overline{D} = (D_1L_1 + D_2L_2)/(L_1 + L_2)$ and where $t_{\mathrm{FWHM}} = (2\ln 2)^{1/2}\tau_1$ is the pulse full width at half-maximum at the midpoint of the first span. At this value of γ, the complete numerical solution of Eq. (1) shows that the pulse shape is nearly Gaussian.[5] The second case, shown in Fig. 2(b), corresponds to $\gamma = 5.83$, at which point the pulse shape differs substantially from a Gaussian.[5] Nevertheless, the variational approach accurately predicts the evolution of the pulse duration and the chirp. The point here is that in the Lagrangian the pulse duration and chirp are defined as integrated characteristics of the pulse through the expressions $\tau = (\int_{-\infty}^{\infty}|q|^2\mathrm{d}t)^2/\sqrt{\pi}\int_{-\infty}^{\infty}|q|^4\mathrm{d}t$ and $\alpha^2\tau^2 + \tau^{-2} = \int_{-\infty}^{\infty}|\partial q/\partial t|^2\mathrm{d}t/\int_{-\infty}^{\infty}|q|^2\mathrm{d}t$. The variational approach integrates over the pulse profile and is to some extent tolerant of the choice of ansatz.

We now apply our approach to determining the dependence of the pulse energy S on γ, keeping D_1, D_2, and $\overline{D}$ fixed and decreasing the full width at half-maximum pulse duration t_{FWHM}. It was previously found empirically that the pulse energy increases quadratically with γ,[4,5] and our results are consistent with that result when γ is small. The quadratic dependence proves to be correct when the average dispersion length $L_{\mathrm{disp}} = t_{\mathrm{FWHM}}{}^2/\overline{D}$ is much larger than the length of the dispersion map, $L_{\mathrm{map}} = L_1 + L_2$. However, when γ increases, the enhancement factor even-

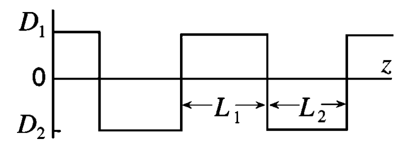

Fig. 1. Schematic illustration of the dispersion map.

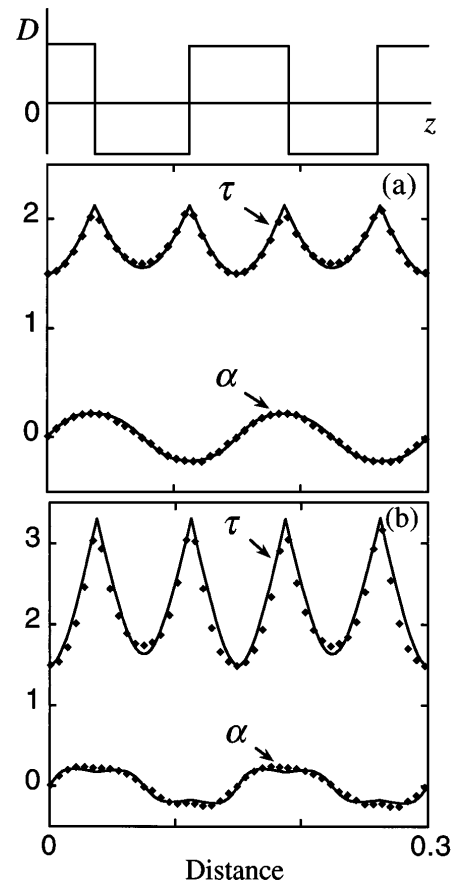

Fig. 2. Comparison of the stable pulse duration and chirp dynamics found with the help of Eqs. (5) and (6) (solid curves) with those produced by direct numerical simulations of Eq. (1) (filled diamonds) in the medium with $\overline{D} = 1$ and a dispersion map length $2L_1 = 2L_2 = 0.1554$; (a) $D_1 - D_2 = 58$, (b) $D_1 - D_2 = 117$.

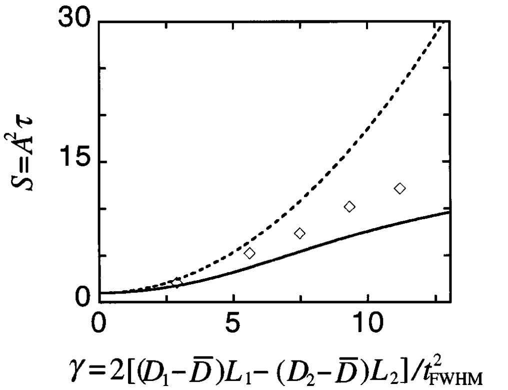

Fig. 3. Dependence of the stationary pulse energy S on γ for the case $L_1 = L_2$, $\overline{D} = 1$, and $D_1 = D_2 = 58$, as predicted in this Letter (solid curve) and in Refs. 4 and 5 (dashed curve). Open diamonds represent the results of direct numerical simulations of Eq. (1).

tually saturates, as shown in Fig. 3. Physically, this saturation occurs when the average dispersion length L_{disp} becomes comparable with the length of the dispersion map L_{map}. Significant discrepancies from quadratic growth in the enhancement factor are visible when $L_{\mathrm{disp}} \lesssim 5L_{\mathrm{map}}$. We verified this saturation by using numerical simulations, although we note, as shown in Fig. 3, that the quantitive agreement with the variational approach is not good. Here both the strength and the weakness of the variational approach are manifest. The computational rapidity and physical transparency of the approach made it possible for us to examine the parameter regime globally and to observe this saturation in the first place. However, its lack of quantitative reliability in the case when γ becomes large and the pulse shape differs significantly from our Gaussian ansatz required us to verify this result, at least in selected instances.

In conclusion, we have shown that the basic characteristics of dispersion-managed soliton dynamics can be successfully described by ordinary differential equations. Focusing on the lossless case, we derived implicit equations for the pulse duration and the chirp, and we showed that these equations can accurately describe the behavior of the dynamics even when the shape differs significantly from a Gaussian. Even under extreme conditions when the approach failed to yield quantitatively accurate results, we found that it still provided a useful starting point for numerical investigations. We note that one can use these equations to determine the required chirp when launching a pulse at the edge of one of the spans of the dispersion map, which was one of our principal motivations for developing this approach. We then applied this approach to determining the dependence of the pulse energy on γ as the pulse duration decreased and showed that the energy saturates.

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