Dispersion-managed solitons at normal average dispersion

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We find that in a dispersion-managed fiber, in which the strength of the dispersion management is above some threshold, solitons can exist with normal average dispersion. When the normal average dispersion is below some limiting value there exist two soliton solutions with the same pulse duration and different pulse energies. When the normal average dispersion is above this limiting value, no soliton exists. Both higher-energy and lower-energy solitons are dynamically stable in the parameter range that we considered. © 1998 Optical Society of America

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It was recently shown that in strongly dispersion-managed fibers, consisting of a periodic dispersion map with alternating spans of anomalous and normal dispersion, solitons can exist not only with anomalous average dispersion but also with zero and normal average dispersion.[1] Here we find, based on a reduced model that we described elsewhere,[2] that if the dispersion difference ΔD between the anomalous and the normal dispersion spans is higher than some threshold, the dependence of soliton energy S on average dispersion $\bar{D}$ is described by a C-shaped curve. The curve starts at $\bar{D} = 0$ when $S = 0$. From there, as S increases, the curve moves into the normal dispersion region but then bends around after reaching a limiting value of normal dispersion, ultimately moving into the anomalous dispersion region. We used a reduced model because it allowed us to explore rapidly a large parameter space, but we checked our results by full numerical simulations in key instances and found good agreement.

Our starting point is the nonlinear Schrödinger equation in the lossless medium, modified to include a spatially varying dispersion $D(z)$:

$$i\,\frac{\partial q}{\partial z} + \frac{1}{2}\,D(z)\,\frac{\partial^2 q}{\partial t^2} + |q|^2 q = 0\,, \tag{1}$$

where we have used a standard normalization of the nonlinear Schrödinger equation. We choose a time scale T_0, which is of the order of a pulse duration, and a (negative) dispersion scale β_2, which is of the order of typical average dispersion in our system. Then we set z as the distance normalized to length scale $L_D = -T_0{}^2/\beta_2$, t as time normalized to T_0, D as dispersion normalized to β_2, and $q = E\,(n_2 \omega L_D/A_{\text{eff}} c)^{1/2}$, where E is the actual field amplitude, n_2 is the Kerr coefficient, ω is the frequency of light, A_{eff} is the effective area, and c is the speed of light. First we analyze this system based on a reduced model obtained with the help of a variational approach.[2] This approach was first described in the context of optical fiber solitons by Anderson[3] and further developed for dispersion-managed solitons in Refs. 4 and 5. We recall that with the ansatz

$$q = A \exp\!\left[\left(-\frac{1}{\tau^2} + i\alpha\right)t^2 + i\sigma\right], \tag{2}$$

where A, τ, α, and σ indicate the pulse's amplitude, duration, chirp, and phase, respectively, the variational method yields the following equations[2–5]:

$$\frac{\mathrm{d}\tau}{\mathrm{d}z} = 2D\alpha\tau\,,$$
$$\frac{\mathrm{d}\alpha}{\mathrm{d}z} = 2D\left(\frac{1}{\tau^4} - \alpha^2\right) - \frac{S}{\tau^3}\,, \tag{3}$$

where $S = U_0/\sqrt{2}$ and $A^2\tau = U_0$ is constant as a function of z. We consider a dispersion map consisting of the anomalous and normal dispersion spans $L_{1,2}$ with dispersions $D_{1,2}$, respectively, periodically repeating in z. Using symmetry and antisymmetry conditions for τ and α, respectively, which imply that $\tau(L_1) = \tau(L_1 + L_2) = \tau_0$ and $\alpha(L_1 + L_2) = -\alpha_0(L_1) = \alpha_0$, one can derive the following eigenvalue equations, which determine the parameters of the dispersion-managed soliton[2]:

$$\frac{f_1}{\sqrt{C_1}} \ln\!\left(\frac{C_1\tau_0 + f_1 + \sqrt{C_1}\,\alpha_0\tau_0{}^2}{\sqrt{f_1{}^2 + C_1}}\right) = C_1 D_1 L_1 + \alpha_0\tau_0{}^2,$$
$$\frac{f_2}{\sqrt{C_2}} \ln\!\left(\frac{C_2\tau_0 + f_2 - \sqrt{C_2}\,\alpha_0\tau_0{}^2}{\sqrt{f_2{}^2 + C_2}}\right) = C_2 D_2 L_2 - \alpha_0\tau_0{}^2, \tag{4}$$

where $f_{1,2} = f(D_{1,2})$, $C_{1,2} = C(D_{1,2})$, $C = \alpha_0{}^2\tau_0{}^2 + 1/\tau_0{}^2 - 2f/\tau_0$, and $f = S/2D$. The pulse durations at the midpoints of the spans with dispersion D_1 and D_2 are then defined, respectively, as

$$\tau_{1,2} = \frac{1}{C_{1,2}}\left(-f_{1,2} + \sqrt{f_{1,2}{}^2 + C_{1,2}}\right). \tag{5}$$

Using Eqs. (4), we vary the average dispersion $\bar{D} = (D_1L_1 + D_2L_2)/(L_1 + L_2)$ with a fixed dispersion difference $\Delta D = D_1 - D_2$ and numerically find the normalized pulse energy S. Figure 1 shows the variation of $S(\bar{D})$ for six values of ΔD when $L_1 = L_2 = 0.0777$. This length corresponds to 100 km when the dispersion unit is $\beta_2 = -0.1\ \text{ps}^2/\text{km}$ and the time unit T_0 is chosen such that the pulse duration is $T_{\text{FWHM}} = 1.763 \times T_0 = 20$ ps. Then $L_D = 1287$ km, and for $A_{\text{eff}} = 47\ \mu\text{m}^2$ and

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$n^2 = 2.6 \times 10^{-16}$ cm^2/W the nonnormalized pulse energy is $0.007 \times S$ pJ. One can see that at small ΔD the curve $S(\bar{D})$ is almost a straight line, and there is no solution with normal average dispersion. However, when ΔD exceeds a threshold value of ~ 190 the curve makes a bend through the normal dispersion regime. Beyond the threshold dispersion difference, a nontrivial solution exists for the dispersion-managed soliton with exactly zero average dispersion.[1] One can see from Fig. 1 that, if the normal average dispersion does not lie below the turning point, there exist two solitons with energies A and B. At the turning point the lower- (B) and higher- (A) energy solitons merge. We verified these results, using complete numerical solutions of Eq. (1). Figure 2 shows the actual FWHM pulse duration at the point of maximum compression in the anomalous dispersion span as a function of normalized pulse energy S along the curve corresponding to $\Delta D = 220$ in Fig. 1. The pulse duration differs by $\sim 9\%$ from what was predicted by the solutions of Eqs. (4), but it remains almost constant at ~ 1.97. We conclude that for every value of average normal dispersion $\bar{D}$ above the turning point there are in fact two solitons with nearly equal pulse durations and different pulse energies. The pulse shapes of the higher- and the lower-energy solitons at the point of maximum compression remain nearly the same, with a flat top and weakly oscillating wings that are apparent only on a logarithmic scale.

A careful study of the evolution during one period of the dispersion map yields an important clue to why strongly dispersion-managed solitons can exist with average normal dispersion. In Fig. 3 we show the evolution of the soliton corresponding to point A in Fig. 1. It is apparent that the soliton undergoes more compression in the anomalous dispersion span than in the normal dispersion span. Consequently, the length of the anomalous dispersion span measured relative to the dispersion length can be greater than that of the normal dispersion span, so the rescaled average dispersion would be negative rather than positive, giving the dispersion and the nonlinearity a chance to balance.

Finally, we consider the stability of the solitons by injecting an initially hyperbolic secant pulse and allowing it to evolve. Figure 4 shows the evolution of a hyperbolic secant pulse to the lower-energy soliton, which corresponds to point B in Fig. 1. One can see the buildup of the oscillating wings of the soliton and then stable propagation of the soliton over a long distance $z = 1000$. For the system parameters indicated above this distance corresponds to more than 1.2×10^6 km. Lower-energy, dispersion-managed solitons were stable even with small energy in the normal average dispersion near the zero-dispersion point when we varied the energy in the range indicated by diamonds in Fig. 2 along the curve corresponding to $\Delta D = 220$ in Fig. 1. This point is significant because Carter *et al.*[6] recently showed that the main source of errors in a dispersion-managed soliton system is the growth of spontaneous-emission noise. The stability of the lower-energy soliton solution may indicate that the noise growth is nonlinearly damped in the normal dispersion regime by being incorporated into the lower-energy soliton. However, the question of soliton stability in the whole parameter space in the normal dispersion regime still remains open. One interesting effect here is a very long-term instability indicated recently by Turitsyn[7] and by Kodama *et al.*,[8] assuming

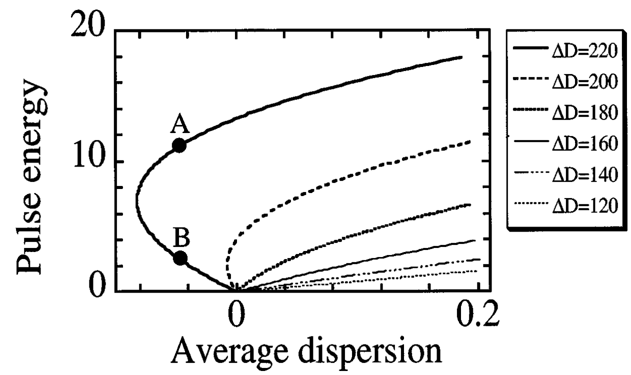

Fig. 1. Dependence of the pulse energy on the average dispersion for six dispersion differences ΔD predicted by the reduced model.

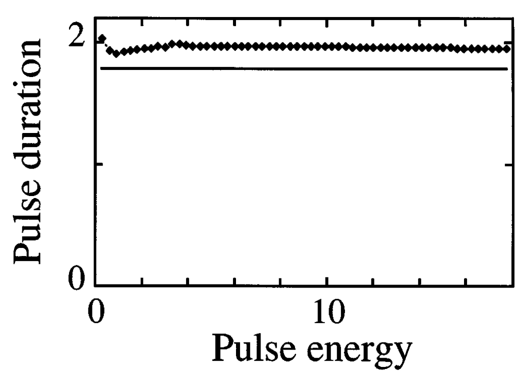

Fig. 2. Comparison of the FWHM pulse durations obtained from the reduced model (solid line) and full simulations (filled diamonds) corresponding to curve $\Delta D = 220$ shown in Fig. 1.

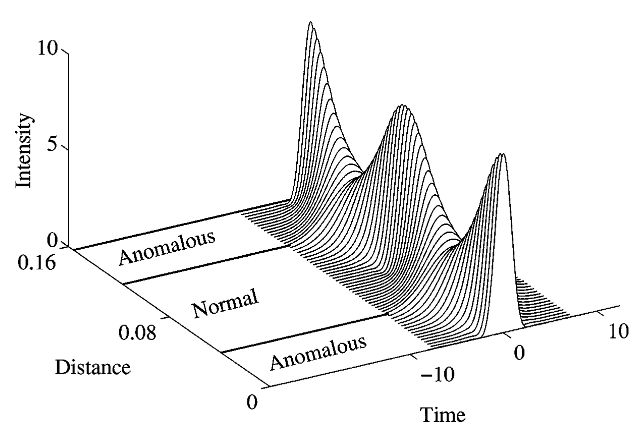

Fig. 3. Pulse evolution of the stable soliton corresponding to point A of Fig. 1 inside one period of the dispersion map. The starting point is the point of maximum compression inside the anomalous dispersion span.

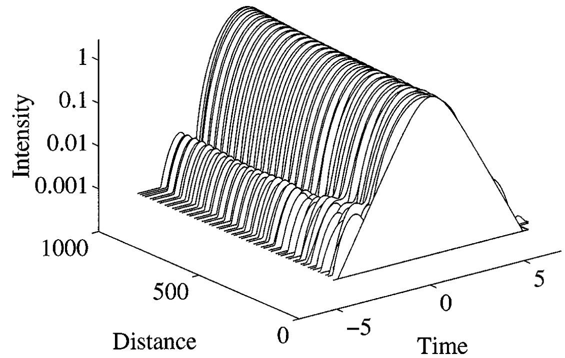

Fig. 4. Evolution of a hyperbolic secant pulse to the stable lower-energy soliton corresponding to point B of Fig. 1. The intensities are cut off at 5×10^{-4}.

a quadratic chirp, that is of conceptual interest even if it might be unobservable in practice because of the intervention of other effects.

In conclusion, we have shown that if the strength of the dispersion management is above some threshold then two different dispersion-managed solitons of higher and lower energy and the same pulse duration can exist when the magnitude of normal average dispersion is below a limiting value. If the magnitude of normal average dispersion exceeds this limit then no dispersion-managed soliton can exist. Both higher- and lower-energy dispersion-managed solitons are dynamically stable, periodically stationary pulses in the range of parameter space that we have considered.

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