Reduced model of the evolution of the polarization states in wavelength-division-multiplexed channels

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We have developed a reduced model of the evolution of the polarization states of the channels in a wavelength-division-multiplexed system that follows only the Stokes parameters for each channel. We apply this model to demonstrating that the expected repolarization of polarization-scrambled signals is small. We verify our results by comparing them with numerical simulations with realistic parameters. © 1998 Optical Society of America

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Previously[1] it was shown that polarization-dependent loss can account for repolarization of a polarization-scrambled signal. Polarization scrambling is used in current long-distance undersea systems to avoid fading,[2,3] and repolarization can lead to renewed fading, which is a significant practical problem. Our previous research was based on a reduced model in which we followed the Stokes parameters of an individual channel and neglected the intersymbol interference that is due to the Kerr nonlinearity and chromatic dispersion. The accuracy of results obtained with this approach is open to question, particularly in a wavelength-division-multiplexed (WDM) system in which neighboring channels can interfere. We developed it because following the complete temporal evolution of many WDM channels often requires prohibitive amounts of computer time. Moreover, the various polarization-dependent loss elements have random orientations, and one must therefore consider a large number of different cases to determine the probability distribution of the repolarization. Following the complete temporal evolution of even a single channel for a large number of cases can also require prohibitive amounts of computer time and for many WDM channels is clearly impractical. Thus there is a real need for well-verified reduced models.

In this Letter we validate the reduced approach just outlined in which we follow the evolution of the Stokes parameters of each WDM channel rather than their full temporal evolution. First we show analytically that this model is applied exactly in the limit of strong dispersion management and derive the equations that govern the nonlinear evolution when polarization mode dispersion (PMD) can be neglected. Intuitively this reduced model works because in the presence of large chromatic dispersion the bits in any channel pass rapidly through the bits of the others, so the bits in one channel are affected only by the Stokes parameters of the other channels. We find an exact solution for the nonlinear equations that govern the polarization evolution, and we show that there is no change in the degree of polarization. This result is significant for long-distance, undersea telecommunication systems because it indicates that the occasional repolarization that is observed in polarization-scrambled signals is not due to chromatic dispersion and nonlinearity and is therefore almost certainly due to polarization-dependent loss.[1] Finally, we verify our reduced approach by comparison with complete simulations, neglecting PMD. We show for realistic parameters that this reduced approach yields reasonable agreement with the simulations and in particular that the repolarization is small.

Our starting point is the Manakov equation[4,5]

$$i\,\frac{\partial \mathbf{U}}{\partial z} - \frac{\beta''}{2}\,\frac{\partial^2 \mathbf{U}}{\partial t^2} + \gamma(\mathbf{U}^\dagger \mathbf{U})\mathbf{U} = 0\,, \qquad (1)$$

where $\mathbf{U} = (u_x, u_y)^t$ represents the complex envelopes of the two polarizations, β'' is the dispersion coefficient, $\gamma = (8/9)(k_0 n_2/A_{\mathrm{eff}})$ is the nonlinear coefficient, and z and t are distance along the optical fiber and retarded time, respectively. Earlier experimental[6] and theoretical[4,5] work showed that Eq. (1) accurately describes nonlinear and dispersive pulse propagation in standard communication fiber with rapidly and randomly varying birefringence when the usual linear PMD can be neglected. In communication systems the residual nonlinear contribution beyond what is included in the Manakov equation, referred to as nonlinear PMD, is completely negligible.[4–6] We also neglect the effect of spatially varying loss and gain, assuming that it occurs on a length scale that is short compared with the nonlinear and dispersive length scales, so its effects can be averaged. Writing $\mathbf{U}$ as a sum of contributions over n channels, we find that

$$\mathbf{U} = \sum_{m=1}^{n} \mathbf{U}^{(m)} \exp[ik^{(m)}z - i\omega^{(m)}t]\,, \qquad (2)$$

where $k^{(m)}$ and $\omega^{(m)}$ are the central wave number and frequency of the mth channel, respectively, and $\mathbf{U}^{(m)}$ is the corresponding wave envelope. Substituting Eq. (2) into Eq. (1), we find that

$$i\,\frac{\partial \mathbf{U}^{(m)}}{\partial z} - \frac{\beta''}{2}\,\frac{\partial^2 \mathbf{U}^{(m)}}{\partial t^2} + \gamma[\mathbf{U}^{\dagger(m)}\mathbf{U}^{(m)}]\mathbf{U}^{(m)} + \gamma \sum_{q=1,\,\neq m}^{n} [\mathbf{U}^{\dagger(q)}\mathbf{U}^{(q)}]\mathbf{U}^{(m)} + \gamma \sum_{q=1,\,\neq m}^{n} [\mathbf{U}^{\dagger(q)}\mathbf{U}^{(m)}]\mathbf{U}^{(q)} = 0\,, \qquad (3)$$

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where, consistent with our assumption that the dispersion between channels is large, we neglect the four-wave mixing terms. We now define the Stokes parameters for each of the channels as

$$S_0^{(m)} = \frac{1}{T}\int_{t_1}^{t_2}[|u_x^{(m)}(t)|^2 + |u_y^{(m)}|^2]\,\mathrm{d}t\,, \tag{4a}$$

$$S_1^{(m)} = \frac{1}{T}\int_{t_1}^{t_2}[|u_x^{(m)}(t)|^2 - |u_y^{(m)}|^2]\,\mathrm{d}t\,, \tag{4b}$$

$$S_2^{(m)} = \frac{2}{T}\int_{t_1}^{t_2}\mathrm{Re}[u_x^{(m)}(t)u_y^{(m)*}(t)]\,\mathrm{d}t\,, \tag{4c}$$

$$S_3^{(m)} = \frac{2}{T}\int_{t_1}^{t_2}\mathrm{Im}[u_x^{(m)}(t)u_y^{(m)*}(t)]\,\mathrm{d}t\,, \tag{4d}$$

where we assume that $T = t_2 - t_1$ contains a large number of individual bits. Using Eq. (3) to determine the evolution of the Stokes parameters, we find that $\mathrm{d}S_0^{(m)}/\mathrm{d}z = 0$ and we find for $\mathrm{d}S_1^{(m)}/\mathrm{d}z$ that

$$\begin{aligned}\frac{\mathrm{d}S_1^{(m)}}{\mathrm{d}z} = i\,\frac{\gamma}{T}\int_{t_1}^{t_2}\Big\{&[u_x^{(m)}u_y^{(m)*} + u_x^{(m)*}u_y^{(m)}]\\ &\times\sum_{q=1,\neq m}^{n}[u_x^{(q)}u_y^{(q)*} - u_x^{(q)*}u_y^{(q)}]\\ &-[u_x^{(m)}u_y^{(m)*} - u_x^{(m)*}u_y^{(m)}]\\ &\times\sum_{q=1,\neq m}^{n}[u_x^{(q)}u_y^{(q)*} + u_x^{(q)*}u_y^{(q)}]\Big\}\,\mathrm{d}t\,. \end{aligned}\tag{5}$$

In a highly dispersive system, the channels for which $q \neq m$ rapidly pass through the mth channel in the time domain. Consequently the evolution of the mth channel is affected only by the averaged time variation of the $q \neq m$ channels, so we can effectively treat them as continuous waves. We thus replace

$$[u_x^{(q)}u_y^{(q)*} - u_x^{(q)*}u_y^{(q)}] \longrightarrow \frac{1}{T}\int_{t_1}^{t_2}[u_x^{(q)}u_y^{(q)*} - u_x^{(q)*}u_y^{(q)}]\,\mathrm{d}t\,, \tag{6}$$

from which we conclude that

$$\frac{\mathrm{d}S_1^{(m)}}{\mathrm{d}z} = \gamma\sum_{q=1}^{n}[S_2^{(m)}S_3^{(q)} - S_3^{(m)}S_2^{(q)}]\,. \tag{7}$$

We can find similar expressions for $\mathrm{d}S_2^{(m)}/\mathrm{d}z$ and $\mathrm{d}S_3^{(m)}/\mathrm{d}z$, so we finally obtain

$$\frac{\mathrm{d}\mathbf{S}^{(m)}}{\mathrm{d}z} = \gamma\mathbf{S}^{(m)} \times \sum_{q=1}^{n}\mathbf{S}^{(q)}\,. \tag{8}$$

The effect of dispersion does not appear in Eq. (8); only the nonlinearity appears, and the equations are analogous to the equations that govern nonlinear polarization rotation of continuous-wave beams.[7] However, the local dispersion is critical because it must be large enough that each channel appears as a continuous-wave background to its neighbors. It is an immediate consequence of Eq. (8) that the Stokes parameters of a single channel do not evolve. Moreover, regardless of the number of channels, the polarization of each channel simply rotates, so the degree of polarization is not changed. In particular, a polarization-scrambled channel cannot repolarize.

Although Eq. (8) is nonlinear, a complete analytical solution can be found. This result is intrinsically significant because the number of large-dimensional nonlinear systems for which exact solutions can be found is small; however, the form is somewhat cumbersome and is not presented here.

We studied the effectiveness of the Eq. (8) by simulating nonreturn-to-zero signal transmission with dispersion management. We polarization scramble our signals, using synchronous phase modulation as discussed by Bergano *et al.*[2] We also use synchronous phase modulation and amplitude modulation to minimize the distortion of the nonreturn-to-zero signal. Polarization scrambling of the optical carrier is achieved by differential modulation of the optical phases of two orthogonal polarization states with a sinusoidal signal, $u_x^{(m)}(t) = A_x(t)\exp[i\phi_x(t)]\cos(\Omega t/2)$ and $u_y^{(m)}(t) = A_y(t)\,\exp[i\phi_y(t)]\cos(\Omega t/2)$, where $\phi_x(t) = \delta_x + a_x\cos(\Omega t + \psi)$ and $\phi_y(t) = \delta_y + a_y\cos(\Omega t + \psi)$. Here we let $A_x(t) = c_xH(t)$ and $A_y(t) = c_yH(t)$, where $H(t) = 1$ in the time slots of the one bits and $H(t) = 0$ in the time slots of the zero bits; c_x and c_y are constant coefficients. a_x and a_y are two phase modulations. We chose the values $a_x = 3.307$ and $a_y = 0.903$, so the difference $a_x - a_y$ nearly equals $j_{0,1} = 2.405$, the first zero of the zeroth Bessel function. With this choice and with the setting $c_x = c_y$, an ideal square pulse would be depolarized. The sum was chosen to be consistent with the experiments of Bergano *et al.*[2] The phase modulation frequency Ω corresponds to the bit rate, ψ describes the relative phase between the phase modulation and the data bits, and δ_x and δ_y denote arbitrary offsets. By varying ψ, δ_x, δ_y, c_x, and c_y we can adjust the degree of polarization to any desired value.

We studied multichannel systems at 5 and 10 Gbit/s. The pulse streams have a path-averaged power of 0.4 mW. Typical experimental dispersion maps use a span of normal dispersion fiber at $D_1 = -2$ ps/nm km followed by a span of anomalous dispersion fiber at $D_2 = 17$ ps/nm km, with a total dispersion of zero.[1] To make the dispersion management stronger for comparison with the theory, we often multiplied the values of D_1 and D_2 by constant factor up to 10. The map length was 200 km. For a 1000-km long map we obtained even better agreement with theory.

For a two-channel 10-Gbit/s WDM system with a 1-nm channel spacing we found that, when $D_1 = -20$ ps/nm km and $D_2 = 170$ ps/nm km, the Stokes vectors of both channels evolve exactly as predicted by Eq. (8), as shown in Figs. 1(a) and 1(b). Even when we decreased the values of D_1 and D_2 to $D_1 = -2$ ps/nm km and $D_2 = 17$ ps/nm km, the Stokes vectors did not change much from the theoretical predication shown in Fig. 1(c), although the difference is

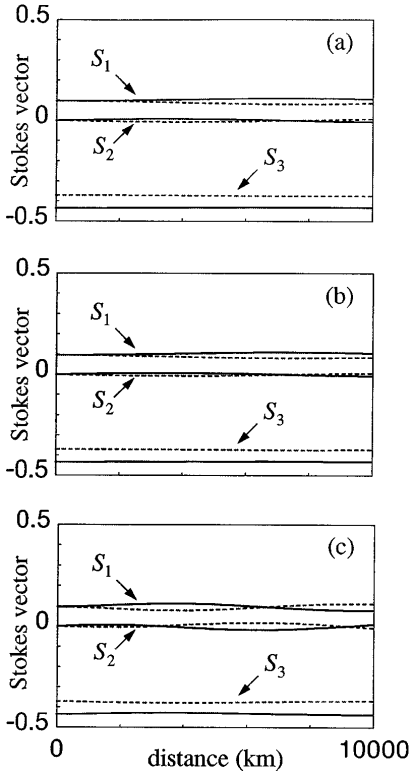

Fig. 1. Evolution of the Stokes vector components as a function of distance. Solid curves, the Stokes components of channel 1; dashed curves, the Stokes components of channel 2. (a) Analytical result. (b) Simulation result: $D_1 = -20$ ps/nm km, $D_2 = 17$ ps/nm km. (c) Simulation result: $D_1 = -2$ ps/nm km, $D_2 = 170$ ps/nm km. Parameter values: $\delta_x = 0$ and $\delta_y = 0$ for both channels; $\psi = 1.5$ for channel 1 and $\psi = 0.8$ for channel 2; path-averaged power in the x polarization, 0.22 mW, and in the y polarization, 0.2 mW, for both channels.

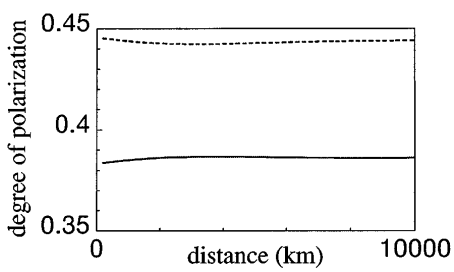

Fig. 2. Evolution of the degree of polarization of two channels as a function of distance with $D_1 = -2$ ps/nm km and $D_2 = 17$ ps/nm km. The parameter values are the same as for Fig. 1.

visible. We also observed that, consistent with theory, the degree of polarization does not change much, as shown in Fig. 2. In particular, if the initial degree of polarization is zero, then the final degree of polarization is less than 0.1. The degree of polarization is defined as $d_{\mathrm{pol}} = [S_1^{(m)^2} + S_2^{(m)^2} + S_3^{(m)^2}]^{1/2}/S_0^{(m)}$. We note that, even though the Stokes vector is small in this case, S_0 is large so the evolution is nonlinear. We used bit patterns that were 64 bits long, and we verified that the results are not sensitive to the exact bit pattern chosen. We also note that, motivated by the experiments of Bergano *et al.*, we focused on a case in which the initial value of the Stokes vector components is small, but our approach also works well in general.

In a two-channel 5-Gbit/s system with 0.5-nm channel spacing we found a similar result, although the repolarization was somewhat larger. Moreover, as we increased the number of channels the agreement with theory improved at both 5 and 10 Gbits/s.

In conclusion, we have developed a reduced approach for modeling the evolution of the Stokes parameters of the channels in a WDM system. We showed that this reduced model is exact in the limit of strong dispersion management. We applied this model to the practically important issue of repolarization of polarization-scrambled signals and showed that the repolarization cannot be caused by chromatic dispersion and nonlinearity alone. We compared our predictions with simulations made with realistic parameters, and we found good agreement with the reduced model.

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