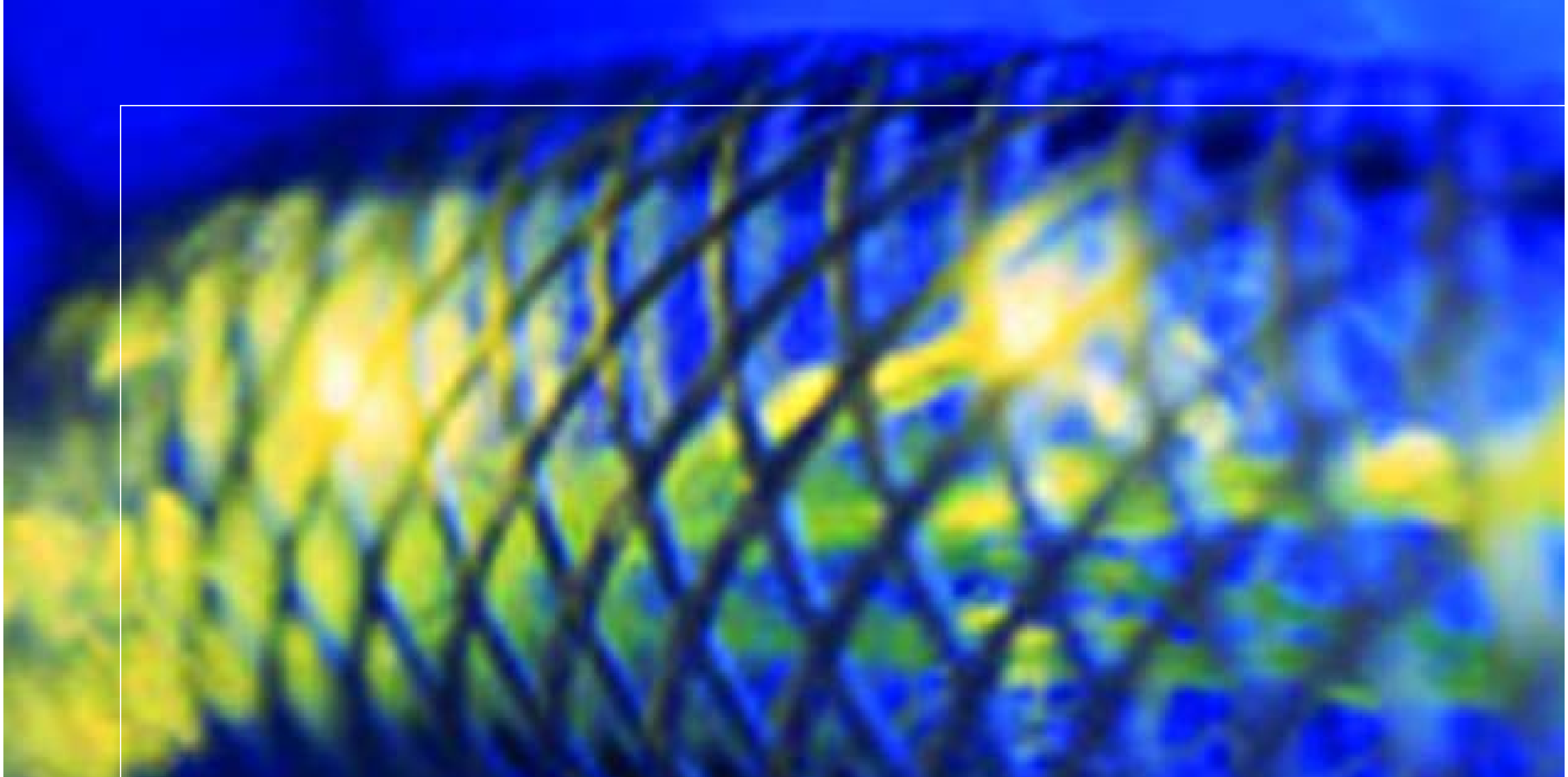

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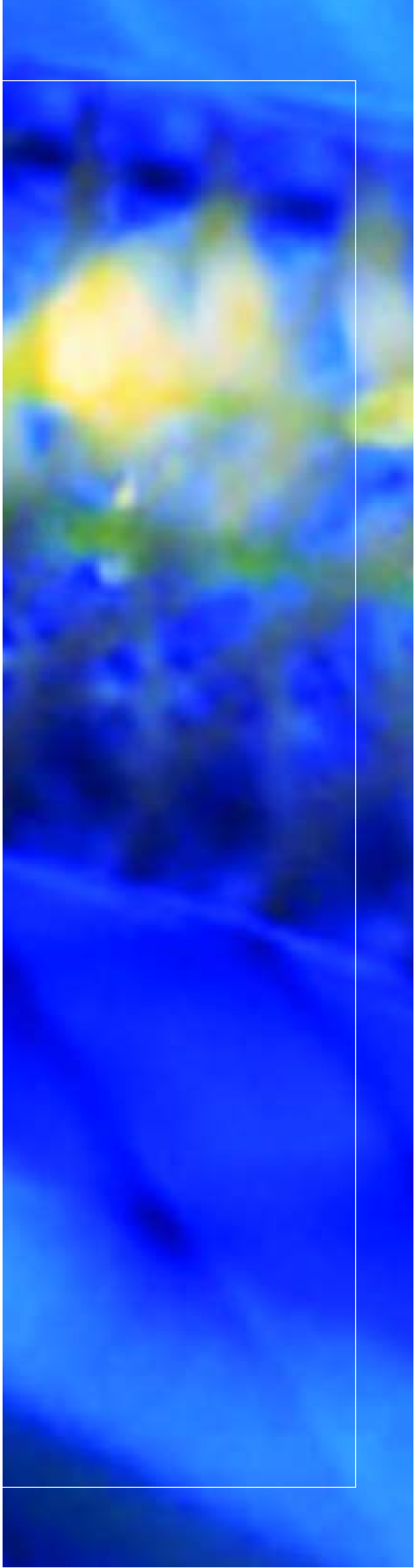
Modeling Receivers in OPTICAL COMMUNICATION SYSTEMS With Polarization Effects

Recent experiments have demonstrated that the bit-error rate of an optical fiber communication system can vary widely due to the random nature of the polarization effects in the system. Using a newly developed receiver model, we show that the bit-error rate depends not only on the optical signal-to-noise ratio but also on the polarization states of the signal and the noise. With the new model we obtained excellent agreement between simulations and experiments for a long-haul transmission system.



There is strong interest in understanding and quantifying how polarization effects influence the performance of optical fiber communication systems. The most widely used performance measures are the optical signal-to-noise ratio (OSNR), the Q-factor and the bit-error rate. These performance measures are progressively more fundamental but also more difficult to measure. Their complex relationship to each other, which is fully understood only in special cases, is significantly complicated by polarization effects. It is particularly difficult to characterize the impact of polarization effects on system performance since they are inherently stochastic in nature because of the random variations in the birefringence orientation and strength of optical fibers. In our laboratory, we emulate long-haul optical fiber communication systems by propagating an optical signal many times around a recirculating optical fiber loop. In these experiments, we observed that the three performance measures listed are not uniquely related to each other and that they can each vary widely depending on the particular realization of the random polarization effects in the loop. How can we explain these performance variations? Given a set of system parameters and a stochastic model for the randomly varying birefringence, can we accurately relate the three performance measures to each other and calculate their probability density functions? To answer these questions, we had to develop accurate models of the transmission line and the receiver that take into account polarization effects.

In the context of this article, the most important polarization effects are polarization-mode dispersion (PMD), which is caused by the rapidly and randomly varying birefringence in optical fiber, and polarization-dependent loss (PDL), which is present in polarization-sensitive devices such as the couplers and isolators in erbium-doped fiber amplifiers. Polarization-mode dispersion causes frequency-dependent random rotations of the polarization state of the light as it propagates through the fiber. As we will explain, in combination with the gain saturation effect in optical amplifiers, PMD and PDL can cause random variations in the OSNR, and hence in the Q-factor and the bit-error rate. To calculate the variation in the OSNR, Wang and Menyuk[1] developed a reduced transmission model for the polarization effects in single-channel and wavelength-division multiplexed systems. But to accurately relate the OSNR to the Q-factor and to the bit-error rate, we needed a model of the receiver that takes into account the polarization effects. Because none of the existing receiver models could accurately account for the effect of partially polarized light, we were forced to develop our own model.

The purpose of this article is to describe the new receiver model we developed. We will show how it helped us to obtain excellent agreement between theory and experiment, thereby leading us to a deeper understanding of how the performance of an optical fiber communication system is influenced by polarization effects. One of the main predictions of our model is that, even if two random realizations of the polarization effects in a system produce the same OSNR, they can nevertheless result in very different bit-error rates. As we will demonstrate, the reasons are that in the transmission line the amplified spontaneous emission noise generated by the optical amplifiers can become partially polarized because of PDL, and that in the receiver, both the Q-factor and the bit-error rate depend not only on the OSNR but also on the polarization state of the noise.

Prior work on receiver modeling

To put our results in context, we begin with a brief review of prior work on receiver modeling for modern long-haul optical fiber communication systems. In these systems, the optical noise entering the receiver dominates the electrical noise generated in the receiver. The optical noise is produced by erbium-doped or Raman fiber amplifiers along the transmission line and by an optical preamplifier just prior to the receiver. The performance of these systems is primarily determined by the shape of the noise-free or noise-averaged signal, the statistics of the noise immediately prior to the receiver, and how the optical filter, square-law photodetector, and low-pass electrical filter in the receiver shape the signal and filter the noise to produce the electrical current. Other important parameters that affect the performance are the sampling time and the decision level used to differentiate between the marks (ONEs) and spaces (ZEROs). The bit-error rate is determined by the probability density functions (pdfs) of the sampled electrical currents of the marks and spaces. One can compute the bit-error rate by use of standard or biasing Monte Carlo simulations to obtain histograms of the currents in the marks and spaces, where different samples correspond to different realizations of the random variables in the system. These variables could include the noise added at each frequency by each of the optical amplifiers, the binary digit in each bit slot, and the variables used to specify the randomly varying birefringence of the optical fiber. However, since bit-error rates are typically on the order of 10^{-9} or less, it is not practical to compute them by use of standard Monte Carlo simulations. Even if we used efficient biasing Monte Carlo methods such as importance sampling or the multicanonical Monte Carlo technique of Berg and Neuhaus, we would need on the order of 10^5 system realizations to accurately compute the bit-error rate for a single set of system parameters. Although this number of realizations is feasible for validation studies, it is too large for the parametric studies required for system design and performance evaluation.

For this reason, despite the recent impressive successful use of biasing Monte Carlo methods to calculate bit-error rates, it is still necessary to use reduced deterministic methods that are as accurate as possible and that only require the user to propagate data through the system once or at most a few times. One must validate these deterministic models and determine their range of applicability by comparison to Monte Carlo simulations and to experiments. The simplest deterministic models consider only the effect on the bit-error rate of noise from optical amplifiers. These models ignore pattern dependences caused by nonlinear pulse-to-pulse interactions, nonlinear interactions between the signal and the noise, and random polarization effects. The behavior of such a system is equivalent to that of a back-to-back system consisting of a transmitter, an optical pre-amplifier that adds unpolarized white Gaussian noise to the signal, and a receiver. In this situation, the appropriate inputs to the receiver model are the shape of the noise-free signal and the noise spectral density or, equivalently, the OSNR.

In the early 1990s, Marcuse[2] and Humblet and Azizoğlu[3] showed that, for a receiver that consists of a rectangular bandpass optical filter, a square-law photodetector and an integrate-and-dump electric filter, the pdfs of the marks and spaces are non-central χ^2-distributions. The χ^2-distribution, which is the pdf of a sum of squares of independent Gaussian random variables with identical variances, arises here because the optical noise is additive white Gaussian noise and the photodetector squares the sum of the signal and the noise. This analysis was extended in various directions by Lee and Shim, Bosco et al., Forestieri and Holzlöhner et al. In particular, they showed that, for arbitrary optical and electrical filters, the pdfs of the marks and spaces are generalized χ^2-distributions.

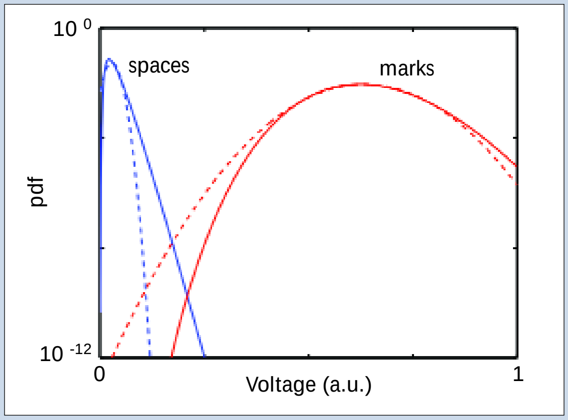


Figure 1. Probability density functions of the marks and spaces at the sampling time of the low-pass filtered received electrical current for a back-to-back system. The solid curves show the exact χ^2 pdfs and the dashed curves show the Gaussian approximations obtained using the means and standard deviations of the exact pdfs.

However, because the numerical calculation of these pdfs is not straightforward, it is often assumed that the electrical currents in the marks and spaces are Gaussian distributed, and so are characterized by their means and variances. In this situation, the bit-error rate is given by

$$\mathrm{BER} = \mathrm{erfc}(Q/\sqrt{2})/2, \qquad (1)$$

where the Q-factor is defined by

$$Q = \frac{\bar{I}_1 - \bar{I}_0}{\sigma_1 + \sigma_0}. \qquad (2)$$

Here $\bar{I}_0$ and $\bar{I}_1$ are the means of the low-pass filtered electrical current at the sampling time for the spaces and marks, respectively, and σ_0 and σ_1 are the corresponding standard deviations. In the case of an integrate-and-dump receiver, the Q-factor can be expressed in terms of the electrical signal-to-noise ratio, $\mathrm{SNR} = (\bar{I}_1 - \bar{I}_0)/\bar{I}_0$, by the formula

$$Q = \frac{\mathrm{SNR}\sqrt{2TB_{\mathrm{opt}}}}{1+\sqrt{1+2\mathrm{SNR}}}, \qquad (3)$$

where T is the bit period and B_{opt} is the bandwidth of the rectangular optical filter.[2] Although this method for estimating the bit-error rate involves a number of approximations, it has been widely used because it enables the bit-error rate to be obtained very easily from the signal-to-noise ratio.

In the past few years, the modeling and analysis of receivers have been reinvigorated by the work of Peter Winzer. One of Winzer's contributions was to derive formulae for the means and variances of the marks and spaces for a receiver with arbitrary optical and electrical filters, under the assumption that the optical noise entering the receiver is unpolarized additive white Gaussian noise.[4] Although these formulae are expressed in terms of multidimensional integrals involving the noise-free signal and the filters, it is easy and fast to compute them numerically.[5] Using Eqs. (1) and (2), the moments of the electrical current can be used to estimate the bit-error rate under the assumption that the current in each bit is Gaussian-distributed rather than χ^2-distributed. It has been shown that, for systems with unpolarized additive white Gaussian noise, over a wide range of system parameters bit-error rates computed using this approach are in good agreement with those obtained using the exact χ^2 pdfs. But as we see in Fig. 1, in the tails, the Gaussian approximation underestimates the χ^2 pdf in the spaces and overestimates it in the marks. Because of this fortuitous cancellation of two errors, the minimum bit-error rates agree with each other to within an order of magnitude, although the optimal voltage thresholds differ by almost a factor of two in this case. Consequently, receiver sensitivities that are calculated using the Gaussian approximations agree well with those obtained using the χ^2 pdfs. The receiver sensitivity is most commonly defined as the power that produces a bit-error rate of 10^{-9}. This result, which is a typical example of a general phenomenon,[2] is for a prototypical back-to-back system. We used a 10 Gbit/s return-to-zero raised cosine signal with an optical signal-to-noise ratio of 20 dB and an extinction ratio in the spaces (ZEROs) of 20 dB. The receiver filters were a 60 GHz Gaussian optical filter and an 8 GHz low-pass fifth-order electrical Bessel filter.

Winzer's deterministic method can also be used to calculate electrical eye diagrams for realistic terrestrial transmission systems. In Fig. 2, we show eye diagrams for a system similar to an experimental loop in our laboratory that we are using to emulate a long-haul terrestrial system with four channels spaced 200 GHz apart.[6] The main nonlinear effects in this system are intrachannel pulse-to-pulse interactions that result in bit-pattern-dependent timing shifts and distortions of the pulses. The eye diagram in Fig. 2 (a) is computed using the deterministic method at a transmission distance of 3,400 km. For this simulation, we propagated the noise-free signal through the system and added the appropriate amount of unpolarized additive white Gaussian noise at the receiver. Therefore, although we correctly modeled nonlinear pulse-to-pulse interactions, we did not model the nonlinear interactions in the fiber between the signal and the noise. In Fig. 2 (b), we show the corresponding result obtained from a Monte Carlo simulation in which we included the nonlinear interactions between the signal and the noise. For a wide range of system parameters, the relative error in the Q-factors computed using these two methods was less than 0.5 percent back-to-back and less than 5 percent for all distances less than 3,400 km. We did not include polarization effects in either simulation.

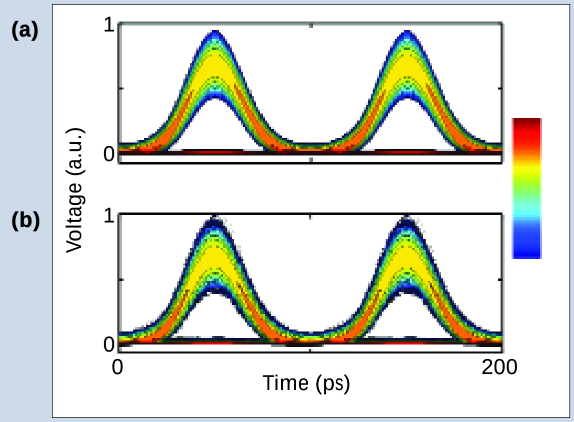


Figure 2. Eye diagrams for a terrestrial optical fiber transmission system at a transmission distance of 3,400 km, calculated (a) using Winzer's deterministic method, with unpolarized, white noise added at the receiver, and (b) using a Monte Carlo simulation that accounts for nonlinear interactions between the signal and the noise in the fiber.

Polarization effects in receivers

Polarization effects cause random variations in both the OSNR and the polarization state of the noise in long-haul systems.[1,7] To understand why, suppose for simplicity that within the bandwidth of a channel the signal is polarized so that its polarization state can be characterized by its Stokes vector, which is a unit-length vector $\mathbf{s} = (S_1, S_2, S_3)/S_0$, where S_0, S_1, S_2, S_3 are Stokes parameters. Suppose that in each PDL element, the Stokes vector of the signal in a given channel is closely aligned with the Stokes vector that has the highest loss in that element. Since the gain saturation of the amplifiers keeps the total power fixed, as the propagation distance increases the signal in this channel will gradually lose power to the other channels and to the noise, and its OSNR will become lower than average. The degree of alignment of the Stokes vector of the signal with the high-loss Stokes vectors of the PDL elements depends on the realization of the PMD in the fibers. Consequently, variations in the PMD result in variations in the OSNR and hence in the bit-error rate.

The average Stokes parameters of the noise in a given bandwidth can be decomposed as the sum of a polarized part of the noise and an unpolarized part. The polarization state of the noise can be characterized by the Stokes vector of the noise, which is a unit-length vector determined by the polarized part of the noise, and the degree of polarization of the noise, which is the power ratio of the polarized part of the noise to the total noise. If, in each PDL element, the Stokes vector of the noise is closely aligned with the Stokes vector with lowest loss, then since the optical amplifiers keep the total power fixed, the noise within the bandwidth of a given channel can become partially polarized by the time it reaches the receiver. Moreover, the degree of polarization of the noise and the angle between the Stokes vectors of the signal and of the noise in a given channel can vary widely depending on the random realization of the PMD.

In the receiver, variations in the polarization state of the noise result in variations in the Q-factor, even if the OSNR is fixed. To explain and quantify this effect, we derived a formula for the Q-factor when the noise is partially polarized.[8] We assumed that, within the bandwidth of the optical filter, the signal is polarized with Stokes vector $\mathbf{s}$ and the noise is partially polarized additive white Gaussian noise that is characterized by the noise spectral density N_{ASE}, the degree of polarization of the noise DOP_n, and the Stokes vector of the noise $\mathbf{n}$. Generalizing Winzer's results, we derived formulae for the mean and standard deviation of the low-pass electrically filtered current in the receiver.

In the derivation of these formulae, care must be taken when the noise is partially polarized, since the components of the noise that are parallel and perpendicular to the signal in Jones space may be correlated with each other due to the combined effect of PMD and PDL. We showed that the mean of the current is independent of the polarization state of the noise. The variance of the current at time t can be expressed as $\sigma_i^2(t) = \sigma^2_{\text{S-ASE}}(t)+\sigma^2_{\text{ASE-ASE}}$, where $\sigma^2_{\text{S-ASE}}(t)$ is the variance caused by the beating of the signal with the noise in the receiver, and $\sigma^2_{\text{ASE-ASE}}$ is the variance caused by the beating of the noise with itself.

We proved that the variance due to signal-noise beating is of the form, $\sigma^2_{\text{S-ASE}}(t) = N_{\text{ASE}}\,\Gamma_{\text{S-ASE}}\,J_{\text{S-ASE}}(t)$, where

$$\Gamma_{\text{S-ASE}} = \tfrac{1}{2}(1 + \text{DOP}_n\,\mathbf{s}\cdot\mathbf{n}), \quad (4)$$

and where $J_{\text{S-ASE}}(t)$ is independent of the noise and depends only on the noise-free signal and the receiver filter shapes. To understand this formula, we observe that, in the receiver, the signal only beats with that portion of the noise that is co-polarized with it. For a fixed noise spectral density, the signal-noise beating variance is maximized when the noise is totally polarized and is co-polarized with the signal, i.e., $\Gamma_{\text{S-ASE}} = 1$, since in this case all the noise beats with the signal. At the other extreme, the signal-noise beating variance is minimized when the noise is totally polarized and the Jones vectors of the signal and the noise are orthogonal, i.e., $\Gamma_{\text{S-ASE}} = 0$, since in this case there is no beating between the signal and the noise. When the noise is unpolarized $\Gamma_{\text{S-ASE}} = 0.5$ is in the middle.

Similarly, the variance of the current caused by noise-noise beating is of the form $\sigma^2_{\text{ASE-ASE}} = N^2_{\text{ASE}}\,J_{\text{ASE-ASE}}/\Gamma_{\text{ASE-ASE}}$, where

$$\Gamma_{\text{ASE-ASE}} = \frac{1}{1+\text{DOP}_n^2}, \quad (5)$$

and where $J_{\text{ASE-ASE}}$ is independent of the signal and noise and depends only on the receiver filter shapes. The noise-noise beating variance is maximized when the noise is totally polarized, since in this case all the noise beats with itself. The minimum value, which is half the maximum value, occurs when the noise is unpolarized. Putting all this together, the formula for the Q-factor for systems with partially polarized noise is

$$Q = \frac{\xi\,\text{OSNR}(\Gamma_{\text{ASE-ASE}}\,\mu)^{1/2}}{1+(1+2\Gamma_{\text{S-ASE}}\Gamma_{\text{ASE-ASE}}\,\kappa\,\xi\,\text{OSNR})^{1/2}}. \quad (6)$$

The only polarization-dependent parameters in this formula are $\Gamma_{\text{S-ASE}}$ and $\Gamma_{\text{ASE-ASE}}$. The parameters κ and μ depend on the receiver filter shapes and κ also depends on the shape of the noise-free optical signal.[8] The parameter ξ is the enhancement factor, which is equal to the ratio of the noise-free current of the marks at the sampling time to the average optical power.[5] The enhancement factor quantifies how much the combination of the optical pulse shape and the receiver enhances the electrical signal-to-noise ratio relative to the optical signal-to-noise ratio. Although the formula assumes that there is no noise-free current in the spaces, it can be extended to the case of signals with a finite extinction ratio.

Comparison to experiments

To validate the formula and illustrate the dependence of the Q-factor on the degree of polarization of the noise, we performed a back-to-back experiment in which partially polarized noise was added to a return-to-zero signal.[9] We produced partially polarized noise by combining unpolarized noise from one amplifier with polarized noise generated by passing noise from a second amplifier through a polarizer and a polarization controller. The polarization controller was used to vary the angle between the Stokes vectors of the signal and the noise. In addition, we varied the degree of polarization of the noise. In the experiment, the electrical signal-to-noise ratio was held fixed at 11 dB. In Fig. 3, for each value of the degree of polarization of the noise, we use red circles and blue squares, respectively, to plot the maximum and minimum measured values of the Q-factor on a linear scale. The red and blue curves show the corresponding results obtained using Eq. (6). As we increase the degree of polarization of the noise from 0 to 1, we observe an increase in the range of the Q-factor. This result illustrates the significant impact that partially polarized noise can have on the performance of an optical fiber transmission system.

The real test of a receiver model is to combine it with a transmission line model to study the performance of a realistic experimental optical fiber communication system. As a first step in this direction, we performed experiments on a single-channel dispersion-managed soliton recirculating loop system in which we propagated 10 Gbit/s return-to-zero pulses over a distance of 10,000 km.[10] In this highly nonlinear system, the balance between dispersion and nonlinearity ensures that the pulse shape evolves periodically, with one period each round trip of the

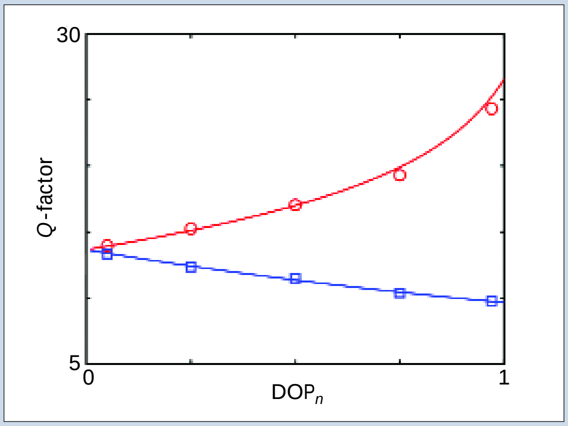


Figure 3. The Q-factor as a function of the degree of polarization of the noise for a back-to-back system. The red curve shows the result obtained using Eq. (6) and the circles show the experimental result when the Jones vectors of the signal and the polarized part of the noise are orthogonal. The blue curve and the squares show the corresponding results when the signal is co-polarized with the polarized part of the noise. [From Ref. 9, Y. Sun et al., Proc. OFC '03.]

107 km loop. Consequently, for at least the first 10,000 km there are essentially no nonlinear interactions between the different pulses, and system performance is primarily determined by the periodically stationary pulse shape, the OSNR, and the polarization states of the signal and the noise. With this system we can therefore study polarization effects independent of other effects. We found that polarization effects in recirculating loops can be very different from those one would expect in a straight-line system. In particular, when we manually varied the setting of a polarization controller in the loop, we observed variations in the bit-error rate that are much larger than would be expected for a comparable straight-line system. These large variations are due to the periodicity of the PMD and PDL in the loop system.

We have been developing and evaluating experimental techniques to make the polarization effects in loops more closely resemble those in a straight-line system, and in particular to reduce the large dependence of the bit-error rate on polarization effects. Inspired in part by the pioneering work of Alan Willner's group at the University of Southern California, we are using a lithium-niobate polarization transformer in the loop to break the periodicity of the polarization effects. This loop-synchronous polarization transformer imparts a different random rotation to the light on each round trip of the loop. To reduce the effect of polarization-dependent gain in this single-channel system, we used a second polarization transformer at the transmitter to scramble the polarization state of the input signal. To study how much the system performance is improved with both polarization transformers, we measured the distribution of the *Q*-factor. With the loop-synchronous polarization transformer, each sample of the *Q*-factor corresponds to a different set of random rotations for each round trip of the loop. In Fig. 4, we show the measured histogram with bars when both polarization transformers were on and the PDL per round trip of the loop was 0.6 dB. The black solid curve shows the corresponding theoretical result obtained by using the new receiver model, together with Wang and Menyuk's reduced transmission line model.[1] This transmission model tracks the evolution of the power and polarization states of the signal and noise, taking into account the polarization effects and the gain saturation of the amplifiers. We have used these results to quantify the extent to which the two polarization transformers improve system performance.[10] Interestingly enough, the *Q*-distributions in Fig. 4 are asymmetric. Our simulations show that this asymmetry occurs because the noise can become partially polarized after many round trips. For those samples for which the signal is often closely aligned with the low-loss axes of the PDL elements, the OSNR is larger than average. But for these same samples, the noise also tends to be co-polarized with the signal. Consequently, the signal-noise beating variance is larger and the *Q*-factor is lower than for a system with the same OSNR but with unpolarized noise. Therefore, the large-*Q* portion of the *Q*-distribution with unpolarized noise is missing, and the *Q*-distribution is asymmetric. To confirm this, we performed a simulation with the same OSNR samples but in which we assumed that the noise was unpolarized at the receiver. As expected, the resulting *Q*-distribution, shown with the blue dashed curve, is much more symmetric. This result illustrates the importance of using a receiver model that accounts for partially polarized noise when modeling systems with PDL.

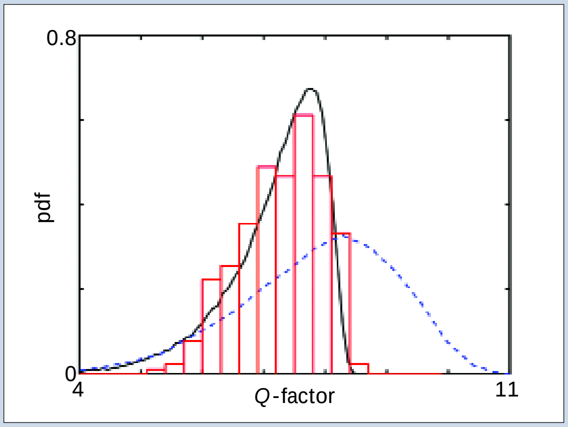


Figure 4. The probability density function of the *Q*-factor for a dispersion-managed soliton recirculating loop at a transmission distance of 10,000 km, where each sample of the *Q*-factor was obtained using a different set of loop-synchronous polarization transformations. The experimental result is shown with red bars and the result obtained using a receiver model that accounts for the polarization state of the noise is shown with a black solid curve. The result obtained using a receiver model that assumes that the noise entering the receiver is unpolarized is shown with a blue dashed curve. [From Ref. 10, Y. Sun et al., IEEE Photon. Technol. Lett. **15**(8), 1067-9 (2003).]

Conclusion

In optical fiber transmission systems with polarization-dependent loss, random variations in fiber birefringence can cause variations in the *Q*-factor and the bit-error rate. By using transmission and receiver models that account for the polarization states of the signal and the noise, we have explained this variation and accurately reproduced the statistics of the performance of an experimental system.

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