Computation of Bit Error Ratios for a Dense WDM System Using the Noise Covariance Matrix and Multicanonical Monte Carlo Methods

Walter Pellegrini, John Zweck, Curtis R. Menyuk, *Fellow, IEEE*, and Ronald Holzlöhner

***Abstract*—We extend the noise covariance matrix method to dense wavelength-division-multiplexed (DWDM) systems in order to efficiently and accurately compute the probability density function of the received voltage in the central channel of a DWDM 10-Gb/s chirped return-to-zero transmission system with a channel spacing of 50 GHz and a transmission distance of 6120 km. The results agree with those that we obtain using a multicanonical Monte Carlo method, which mutually validates both methods.**

***Index Terms*—Amplifier noise, Monte Carlo methods, optical fiber communication, optical Kerr effect.**

I. Introduction

IN OPTICAL transmission systems, the amplified spontaneous emission (ASE) noise that optical amplifiers add to the signal gives rise to bit errors. To compute the bit error ratio (BER), it is typically necessary to compute the probability density functions (pdfs) of the received electrical current in the marks (ones) and spaces (zeros) down to probability density values of 10^{-10} or less. The shape of these pdfs can be affected by nonlinear interactions involving the signal and the noise during transmission. In [1], we introduced a deterministic method called the noise covariance matrix (NCM) method that correctly accounts for nonlinear interactions between the signal and noise, but that neglects any nonlinear interactions of the noise with itself, and we validated the method for a highly nonlinear dispersion-managed soliton system. In [2], we extended the method to quasi-linear single-channel chirped return-to-zero (CRZ) systems, in which pulse overlap during transmission leads to nonlinearly induced bit patterning effects. An important issue is to verify that the method still works for dense wavelength-division multiplexed (DWDM) systems, and to determine how its computational cost scales as the number of channels increases.

In this letter, we extend the NCM method to accurately and efficiently compute the pdfs and the BER for a DWDM CRZ system. We study a five-channel 10-Gb/s per channel system with a 50-GHz channel spacing, transmitting data over 6120 km. This system is similar to the single-channel system in [2], and resembles some commercial DWDM submarine systems. We reduced the number of channels to five since earlier results showed that it is sufficient to simulate a limited number of channels [3].

The extension of the NCM method to several channels is not trivial. For an M-channel system with N discrete frequencies per channel, the NCM is a real $2MN \times 2MN$ matrix. The computational cost of computing the entire covariance matrix is $2MN$ times the cost of propagating the noise-free WDM signal once through the system and scales linearly with the number of channels. Consequently, for a WDM system it is not computationally feasible to compute the entire covariance matrix, even for a few channels. However, we will demonstrate that to accurately compute the pdfs for the central channel of a WDM system, it is only necessary to propagate the covariance matrix of the central channel, which is the $2N \times 2N$ submatrix of the full covariance matrix that represents the correlations between the noise in the frequencies of the central channel. This reduction is physically reasonable, since the correlation between noise at two different frequencies decays proportionally to their frequency separation, and since noise outside the bandwidth of the central channel is mostly filtered out by the optical demuxing filter in the receiver. With this reduction, the cost of the method is $2N$ times the cost of a single simulation of the noise-free WDM system, which no longer scales linearly with the number of channels and is feasible.

We compare our extension of the NCM method to DWDM systems to results obtained using the multicanonical Monte Carlo (MMC) method, which we previously compared to the NCM for a single-channel CRZ system [4]. Because it is a statistical method, all relevant physical effects are properly accounted for in an MMC simulation. However, it is much more computationally costly than the NCM method. The excellent agreement between the NCM and MMC methods provides a mutual validation of both methods for WDM systems of the type we studied.

II. Simulation Procedure

The system we simulated consists of 34 dispersion map periods. Each map period consists of a 10-km span of anomalous dispersion fiber, with a dispersion of $D = 16.5$ ps/(nm $\cdot$ km) at 1551.4 nm, followed by 160 km of normal dispersion fiber with $D = -2.5$ ps/(nm $\cdot$ km), and another 10-km span of the anomalous dispersion fiber. Both fibers have a dispersion

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W. Pellegrini is with the Information Engineering Department, University of Padova, Padova 35131, Italy (e-mail: valter@dei.unipd.it).

J. Zweck is with the Mathematics and Statistics Department, University of Maryland Baltimore County, Baltimore, MD 21250 USA (e-mail: zweck@math.umbc.edu).

C. R. Menyuk is with the Optical Fiber Communications Laboratory, Department of Computer Science and Electrical Engineering, University of Maryland Baltimore County, Baltimore, MD 21227 USA (e-mail: menyuk@umbc.edu).

R. Holzlöhner is with European Southern Observatory, Munich 85748, Germany.

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slope $dD/d\lambda = 0.075$ ps/(nm$^2\cdot$ km). We used precompensation and postcompensation fibers, each of length 9.8 km, with $D = 93.4$ ps/(nm $\cdot$ km) and $dD/d\lambda = 0$. All fibers have a loss of 0.21 dB/km. The loss is compensated by erbium-doped fiber amplifiers (EDFAs) with a spontaneous emission factor of $n_{\rm sp} = 2.0$ that are placed every 45 km, and that restore the power lost in the previous span of fiber.

The envelope of the optical field of a pulse representing a mark is a chirped raised-cosine pulse u with a peak power of 1 mW as in [2]. The pulse representing a space is obtained by multiplying u by the extinction ratio of $\sqrt{25\,\text{dB}}$. The optical signal is polarized and the carrier frequency of the central channel is 1551.4 nm. We transmit five channels with a 50-GHz channel spacing using a 32-bit Debruijn pseudorandom bit sequence. The receiver consists of an optical demultiplexer that demultiplexes out the central channel using a third-order super Gaussian filter with a full-width at half-maximum of 35 GHz, followed by a square law photodetector, and a fifth-order low-pass electrical Bessel filter with a 3-dB bandwidth of 8 GHz.

We now explain how the algorithm in [1] and [2] must be modified to extend the NCM method to WDM systems. Let $u = u_0 + \delta u$ be the optical field envelope, where u_0 is the noise-free signal and δu represents the accumulated noise. We discretize u_0 with $N_{\rm FFT} = 4096$ points. We define the real covariance matrix of dimension $2N$ by $\mathcal{K} = \langle \mathbf{a}\mathbf{a}^\dagger \rangle$, where the dagger denotes the transpose and $\mathbf{a} = (\alpha_{-N/2}, \cdots, \alpha_{N/2-1}, \beta_{-N/2}, \cdots, \beta_{N/2-1})^\dagger$. Here, the quantities α_k and β_k are the real and imaginary parts of the N lowest frequency components of δu. Instead of computing the full NCM representing all pairwise correlations among noise modes within the bandwidth of the WDM signal, we compute only the submatrix of correlations between noise modes within the bandwidth of the central channel. Consequently, we use $N = 160$, corresponding to the 50-GHz bandwidth of the central channel, rather than $N = 5 \times 160$, as would be required for the full covariance matrix. By calculating the full covariance matrix, we verified that the cross correlations between the noise modes in the central channel and those in the other channels are negligible relative to intrachannel noise correlations, and can, therefore, be neglected. However, we still simulate the transmission of the noise-free signal with all five channels, and in the calculation of the covariance matrix of the central channel, we take into account the interaction of the noise in the central channel with the noise-free signal in all channels.

The covariance matrix method is based on the linearization assumption that the noise does not interact with itself during transmission in an appropriate basis set [1]. Previous study of a single-channel CRZ system [2] showed that this assumption breaks down unless we use a basis set in which the phase jitter is separated from the other noise components in each bit. For the results reported here, after each amplifier we artificially demultiplexed the central channel from the other channels before separating the phase jitter for each bit of the central channel. Then, we restored the other channels before continuing the propagation.

To propagate the covariance matrix $\mathcal{K}$ over one fiber span from $z = z_0$ to $z = z_0 + L$, followed by an EDFA with gain G, we use the formula $\mathcal{K}(z_0+L) = G\boldsymbol{\Psi}\mathcal{K}(z_0)\boldsymbol{\Psi}^\dagger + \eta\mathcal{I}$, where $\boldsymbol{\Psi}$ is a $2N \times 2N$ real propagator matrix, $\mathcal{I}$ is the identity matrix, and η is equal to half of the average ASE noise power introduced by the EDFA per frequency mode. We propagate $\boldsymbol{\Psi}$ and extract the phase jitter using the following perturbative algorithm. First, we propagate the noise-free signal u_0 from z_0 to $z_0 + L$ using a standard Fourier split-step algorithm. Then, we return to z_0 and repeat the following for each k:

1) Propagate $u^{(k)}(t, z_0) = u_0(t, z_0) + \epsilon \exp(i\omega_k t)$ through the fiber and EDFA to obtain $u^{(k)}(t, z_0 + L)$. Here, $\epsilon = 0.001$ and ω_k is the frequency of the kth mode of the covariance matrix. The propagation of both $u^{(k)}$ and u_0 is done taking into account all channels. Let $\delta u^{(k)} = u^{(k)} - u_0$.
2) Using a square filter, filter out from $\delta u^{(k)}(t, z_0 + L)$ those frequency components outside the bandwidth of the central channel that may interfere with the phase jitter removal process in the central channel.
3) Separate the pulses in the central channel by passing $\delta u^{(k)}(t, z_0 + L)$ through an ideal lossless fiber with total dispersion equal to $-D(z_0 + L)$, which is the negative of the total accumulated dispersion at $z_0 + L$.
4) Remove the phase jitter as in [2] to give $\widetilde{\delta u}^{(k)}(t, z_0 + L)$.
5) Invert Step 3 by applying an artificial dispersion compensation with a total fiber dispersion equal to $+D(z_0 + L)$.
6) Restore in $\widetilde{\delta u}^{(k)}(t, z_0+L)$ the frequency components that were filtered out in Step 2.
7) Set $\Psi_{jk} = a_j^{(k)}/\epsilon$, where $\mathbf{a}^{(k)}$ is the Fourier transform of $\widetilde{\delta u}^{(k)}(t, z_0 + L)$.

At the receiver, following [1], we compute the pdf of the received electrical signal from u_0 and $\mathcal{K}$ using a version of the method of steepest descents adapted from [5]. Note that if $N_{\rm Ch}$ is the number of frequencies per channel and $N_{\rm Demux}$ is the number of frequencies kept in the artificial demux operation in Step 2, then $N \leq N_{\rm Demux} \leq N_{\rm Ch}$ must hold for a correct computation of the covariance matrix. For, if $N > N_{\rm Demux}$, the phase jitter would not be removed from all of $\mathcal{K}$, while if $N > N_{\rm Ch}$, it would not be removed correctly, since other channels may interfere with the phase jitter removal process.

Next, we briefly outline the MMC method described in [4] that we use to validate the results of the NCM method. To adequately sample the low-probability tails of the electrical pdfs, the MMC method iteratively biases the pdf ρ of the ASE noise. Within each iteration of the method, we use a different biasing pdf $\widetilde{\rho}$ to compute a histogram that approximates the pdf of the voltage V. The histogram of V is defined by discretizing a voltage range into K bins of width ΔV, and the probability that V is in the kth bin is

$$P_k = \int_{\Gamma} \chi_k(\tilde{\mathbf{z}}) L(\tilde{\mathbf{z}}) \widetilde{\rho}(\tilde{\mathbf{z}}) d\tilde{\mathbf{z}}, \qquad 1 \leq k \leq K \tag{1}$$

where $\tilde{\mathbf{z}}$ is a noise realization sampled according to $\tilde{\rho}$, $L(\tilde{\mathbf{z}}) = \rho(\tilde{\mathbf{z}})/\tilde{\rho}(\tilde{\mathbf{z}})$, and χ_k is the indicator function of the subspace Γ_k of the noise probability space Γ defined by $\Gamma_k = \{\tilde{\mathbf{z}} \in \Gamma | (k-1)\Delta V \leq V < k\Delta V\}$. In each iteration, we approximate the integral in (1) by using the Metropolis algorithm to draw samples from $\tilde{\rho}$. The biasing pdf $\tilde{\rho}$ is determined with the aid of a control quantity that is correlated to V. At the end of

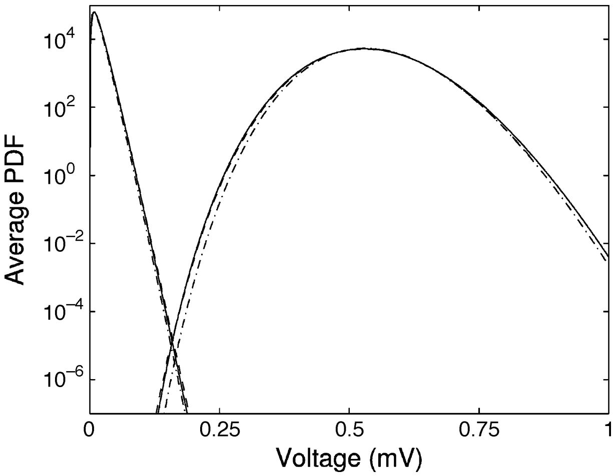

Fig. 1. Average pdfs of the voltage in the marks and spaces for the central channel of a five-channel DWDM CRZ system. The results obtained using the NCM method are shown with solid curves and those obtained using the MMC method are shown with dashed curves. The results obtained using the NCM method for the same system with a single channel are shown with dotted–dashed curves.

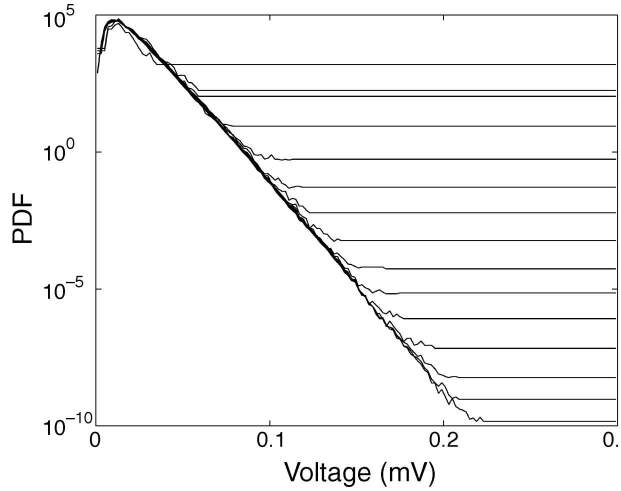

Fig. 2. Convergence behavior of the pdf of the voltage in the first space for 15 iterations of the MMC method. With each additional iteration, the pdf is computed further out on the tail.

each iteration, $\tilde{\rho}$ is updated so that as the number of iterations increases, the number of hits in each bin of the histogram of the control quantity becomes approximately constant, independent of k. In [4], we performed two MMC simulations, one each for the average pdfs in the marks and in the spaces. For this letter, to ensure that the pdfs in each bit converge as the number of iterations increases, we ran 32 MMC simulations to separately compute the pdfs in each bit. For each bit, we chose the control quantity to be the received voltage in that bit.

III. Results

In Fig. 1, we plot the average pdfs of the received voltage in the marks and spaces for the five-channel system using the NCM method (solid curves) and the MMC method (dashed curves). The pdf of each bit is computed at the center of the bit slot. The NCM method required about 5 h of computational time on a Pentium 4 workstation. For each MMC simulation, we produced about 300 000 samples for a mark and 200 000 for a space. The agreement between the two methods is excellent. Integrating the pdfs, we obtain an optimal BER of 6.59×10^{-11} with the NCM method and 1.214×10^{-10} for the MMC method, with threshold levels of 0.1589 and 0.1598 mV, respectively. These small differences may be due to lack of statistical resolution in the MMC method and numerical imprecision rather than to noise–noise interactions. For comparison, we also plot the results for the same system with only a single channel using the NCM method (dotted–dashed curves). In this case, the BER is 1.64×10^{-11} and the threshold level is 0.1635 mV. Note that since we only used 16 bits per channel, we have not accounted for interchannel timing shifts due to all possible pulse collisions. Nevertheless, the agreement between the single- and multiple-channel results shows that the signal in the side channels does not significantly affect the noise in the central channel. In an MMC simulation, a smooth pdf may not be correct. A more reliable way to quickly check for incorrectness is to check whether a pdf converges. In Fig. 2, we show that the pdf in the first space converges as the number of iterations increases. The convergence behavior is similar for all other bits.

IV. Conclusion

We extended the NCM method to WDM systems and applied the method to a 10-Gb/s five-channel DWDM CRZ system. We verified that the correlations between noise in different channels is negligible relative to intrachannel noise correlations. Therefore, the BER of the central channel can be efficiently and accurately obtained from a computation of the submatrix of the covariance matrix corresponding to the frequencies in the central channel. The results obtained are in agreement with those obtained using an MMC method, which mutually validates both methods.

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