Probabilistic model of nonlinear penalties due to collision-induced timing jitter for calculation of the bit error ratio in wavelength-division-multiplexed return-to-zero systems

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We introduce a fully deterministic, computationally efficient method for characterizing the effect of nonlinearity in optical fiber transmission systems that utilize wavelength-division multiplexing and return-to-zero modulation. The method accurately accounts for bit-pattern-dependent nonlinear distortion due to collision-induced timing jitter and for amplifier noise. We apply this method to calculate the error probability as a function of channel spacing in a prototypical multichannel return-to-zero undersea system. © 2006 Optical Society of America

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The effect of optical fiber nonlinearity on light wave transmission is stochastic due to the randomness of the encoded information. As a consequence, it can be difficult to evaluate the impact of the transmission system on the optical signals. In wavelength-division-multiplexed (WDM) systems, a large number of bits in neighboring channels influence a given bit, and the computational effort for a full simulation quickly becomes unacceptably large. A common approach is to treat the nonlinearly induced interchannel interference as additive Gaussian noise.[1–5] Although it has proved to be effective for non-return-to-zero systems, this approach does not take into account collision-induced timing jitter due to cross-phase modulation. Therefore the Gaussian noise approach cannot be directly applied to return-to-zero (RZ) systems, in which timing jitter is the principal effect of fiber nonlinearity.[3,6–8] To account for this timing jitter when calculating the bit error ratio (BER), it is necessary to know the probability density function (pdf) of the time shift of a pulse.[8] By the central limit theorem, it is natural to approximate it with a Gaussian function when the number of pulse collisions is large.

In this Letter, we calculate the pdf of the collision-induced time shift using the Gaussian approximation, and we compare it to the pdf that is obtained using the characteristic function method.[8] We show that while the Gaussian approximation is a poor fit to the true pdf of the time shift, it may result in a reasonably good BER estimate when both collision-induced timing jitter and amplified spontaneous emission noise are present. However, if the channel spacing decreases, leading to a stronger nonlinear cross talk, we will show that one must still use the exact pdf of the time shift. We will provide a physical explanation for this behavior. The work reported here extends an earlier, brief conference report[9] by giving a complete derivation of the basic equations and a discussion of the physics.

To obtain the pdf of collision-induced time shift and the BER, we start by calculating the time-shift function.[10,11] We refer to the pulse u_T for which we are calculating the time shift function as the target pulse and the channel in which it is located as the probe channel. The other WDM channels are referred to as the pump channels. We define the time shift function $\tau(\Delta f_k, l) \equiv \tau_{kl}$ as the time shift of the target pulse u_T after a propagation distance L due to a collision with a pulse u_{kl} that is initially located in the lth bit slot of the kth pump channel, where Δf_k is the frequency offset of the pump channel from the probe channel. We compute this time shift using a semianalytical method presented by Grigoryan and Richter.[7] An important simplifying assumption made in this analysis is that the total time shift of a pulse is the sum of the time shifts caused by individual pairwise collisions. The validity of this assumption is not obvious for modern RZ systems, in which a pulse overlaps with many of its neighbors due to a large dispersive spread. However, this assumption has proved to be valid in both soliton and quasi-linear systems.[7,8] The starting point is a version of the propagation equation, for which the nonlinear part includes only the effects of self-phase modulation and interchannel cross-phase modulation:

$$\frac{\partial u_T}{\partial z} + i\frac{\beta''}{2}\frac{\partial^2 u_T}{\partial t^2} - i\gamma\left(|u_T|^2 + 2\sum_{k,l}\alpha_{kl}|u_{kl}|^2\right)u_T = g u_T, \tag{1}$$

where z is the physical distance, t is the retarded time with respect to the probe channel, β'' is the local dispersion, γ is the nonlinear factor, g is fiber loss and gain coefficient, and $\alpha_{kl}=1$ if the lth bit slot in

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the kth channel contains a pulse, corresponding to a digital 1, and is zero otherwise. While including higher-order dispersion poses no difficulty in principle, its effect on the system under study is negligible, and we set it to zero for simplicity. Defining the central time of the target pulse u_T as $(1/E_T)\int t|u_T|^2 dt$ and its central frequency as $(1/E_T)\int \mathrm{Im}[(\partial u_T^*/\partial t)u_T]dt$, where $E_T=\int|u_T|^2 dt$, we can obtain the total time shift of the target pulse from Eq. (1) as $T_{\text{total}}=\Sigma_{k,l}\alpha_{kl}\tau_{kl}$, where the time shift τ_{kl} is given by

$$\tau_{kl}=\int_0^L \Delta\Omega_{kl}(z)\beta''(z)dz, \quad (2)$$

and $\Delta\Omega_{kl}(z)$ is the collision-induced frequency shift, the evaluation of which is given by

$$\frac{d\Delta\Omega_{kl}}{dz}=-\frac{2\gamma}{E_T(z)}\int_{-\infty}^{\infty}|u_T(z,t)|^2\frac{\partial|u_{kl}(z,t)|^2}{\partial t}dt. \quad (3)$$

When the higher-order dispersion is zero, the pulse shape in all the channels is identical so that

$$u_{kl}(z,t)=u_T\left(z,t-\int_0^z 2\pi\Delta f_k\beta''(x)dx+T_{\text{bit}}l\right), \quad (4)$$

where T_{bit} is the bit period and $l=0$ corresponds to the bit slot of the target pulse.

We assume that the α_{kl} are independent, identically distributed random variables, each having probability 1/2 of being 1 or 0. Thus the total shift of the target pulse T_{total} is a random variable, which is a linear combination of independent binary random variables. The number of collisions is finite but large; so that, based on the central limit theorem, it is reasonable to assume that T_{total} is Gaussian distributed. In this case, it is sufficient to know its mean μ_T and variance σ_T^2. In the absence of higher-order dispersion,[10] since $\tau_{kl}=-\tau_{-k-l}$, we obtain $\mu_T=\Sigma_{k,l}\tau_{kl}=0$ and $\sigma_T^2=\langle T_{\text{total}}^2\rangle=1/4\Sigma_{k,l}\tau_{kl}^2$. Alternatively, the pdf of T_{total} can be computed by using the characteristic function[8] $w(\xi)$, which is given by

$$w(\xi)=\langle\exp(i\xi T_{\text{total}})\rangle=\left\langle\exp\left(i\xi\sum_{k,l}\alpha_{kl}\tau_{kl}\right)\right\rangle, \quad (5)$$

where $\langle\,\rangle$ denotes the statistical average. Since the random variables α_{kl} are independent and the probability that α_{kl} equals either 1 or 0 is 1/2,

$$w(\xi)=\prod_{kl}\left\{\frac{1}{2}[1+\exp(i\xi\tau_{kl})]\right\}. \quad (6)$$

Then the pdf of the time shift is simply the Fourier transform of the characteristic function,

$$p_{T,\text{Char}}(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}w(\xi)\exp(-i\xi t)d\xi. \quad (7)$$

One can in principle accommodate more complex α_{kl}, e.g., pseudorandom, by using the appropriate characteristic function.

We consider a 10 Gbit/s system with a propagation distance of approximately 5000 km.[8] The transmission part includes 100 periods of a dispersion map consisting of 34 km of D_+ fiber and 17.44 km of D_- fiber followed by an amplifier, The values of dispersion, effective core area, nonlinear index, and loss are 20.17 ps/nm km, 106.7 μm^2, 1.7×10^{-20} m^2/W, and 0.19 dB/km for the D_+ fiber and −40.8 ps/nm km, 31.1 μm^2, 2.2×10^{-20} m^2/W, and 0.25 dB/km for the D_- fiber, respectively. The average map dispersion is −0.5 ps/nm km, and the amount of precompensation and postcompensation is 1028 and 1815 ps/nm, respectively. We used 35 ps raised-cosine pulses with a peak power of 5 mW, and we launched nine copolarized channels separated by 50 GHz, each carrying a 32-bit sequence. We verified with both a full simulation model and the reduced models described here that a further increase in the number of channels and number of bits per channel had a negligible effect on the system performance. The receiver included a 30 GHz super-Gaussian optical demultiplexer and a photodetector. We did not consider noise in this part of the study.

Figure 1 shows the time shift pdfs obtained with the Gaussian approximation, the characteristic function method, and importance-sampled (IS) Monte Carlo simulations.[8] The characteristic function method yields a pdf that fits the IS histogram very accurately. This result supports the validity of the assumption that the pulse collisions are additive. In contrast, under the same assumptions, the Gaussian approximation results in a poor fit of the true pdf. The Gaussian function and the IS histogram match closely only near their centers, which is consistent with the central limit theorem. However, since the number of pulse collisions during the propagation is finite, there exists a worst-case time shift so that time shifts that are larger than this maximum time shift have zero probability. As a result, the Gaussian curve is significantly wider than the true pdf in the tails. The full model IS simulations reported here took approximately 500 h to complete, and the reduced model simulations took approximately 10 min to complete on a Pentium IV machine with a 3 GHz processor.

To compute the pdf $p(I,t)$ of the received current I at the sampling time t due to both nonlinear signal distortions and noise, we convolve the pdf p_T of the

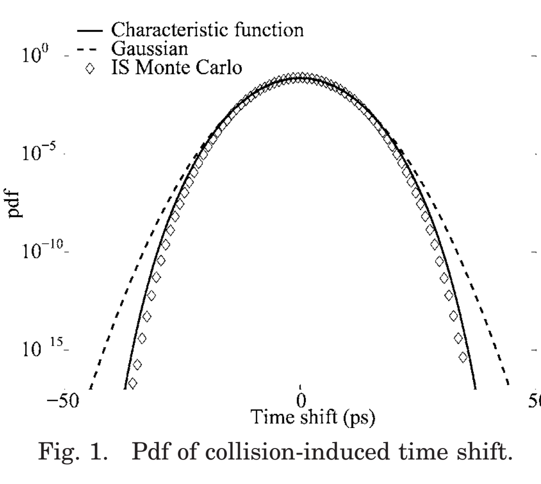

Fig. 1. Pdf of collision-induced time shift.

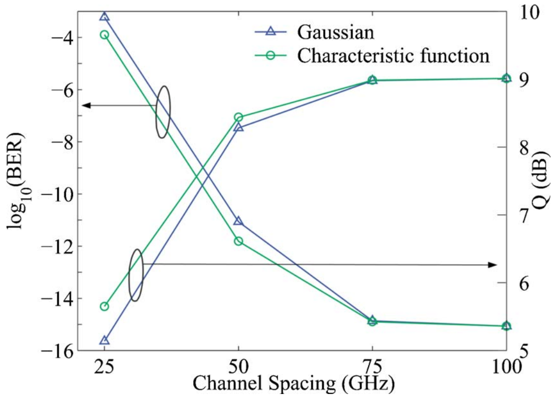

Fig. 2. (Color online) BER as a function of channel spacing.

time shift, using either Eq. (7) or a Gaussian fit with the noise pdf $p_{\text{noise}}(I,t)$ of the received current obtained by propagating a single-channel signal through the system, $p(I,t)=\int p_{\text{noise}}(I,t-\tau)p_T(\tau)d\tau$. To compute $p_{\text{noise}}(I,t)$, we assume that the optical noise is additive white Gaussian noise at the entry to the receiver, and we take into account the actual pulse shape, as well as the frequency-dependent optical and electrical filtering, using an approach described by Forestieri.[12] For the calculation of p_{noise}, we used an 8 GHz electrical fifth-order Bessel filter, and we set the optical signal-to-noise ratio to 15 dB, calculated over a 25 GHz bandwidth.

In Fig. 2, we plot the BER and Q as a function of channel spacing, which was calculated from $p(I,t)$ using the time shift pdfs obtained from the Gaussian approximation and the characteristic function method, Eq. (7). For each point in the plot, the total number of the pump channels was chosen so that they filled a spectral range of 800 GHz around the center frequency of the target channel. As seen in Fig. 2, the difference in BER computed with the two methods is less than an order of magnitude when amplified spontaneous emission noise is present, despite the difference in the time shift pdfs that are obtained with the Gaussian and characteristic function methods. The physical reason for this result is as follows: the pdf of the electrical current $p(I,t)$ is affected mainly by the part of the time shift pdf p_T that has values of 1 down to 10^{-10} to 10^{-12}. Even though there is a significant difference between the Gaussian pdf and $p_{T,\text{Char}}$ in the tails, they agree near their centers. After the convolution procedure, the pdf of the current is changed by weighting with the values of the time shift pdf. Since the Gaussian pdf and $p_{T,\text{Char}}$ are close near their centers, the difference between the two on average is smaller than it is in the tail. As a result, the BER values obtained with the two methods are much closer than the tails of the two time shift pdfs. However, in the case of 25-GHz-spaced channels, the timing jitter leads to a very large error rate. The relative difference in BER calculated with the two methods is much more significant in this case. This difference cannot be neglected when applying forward error-correction (FEC) techniques.

We have characterized the effect of collision-induced timing jitter in WDM RZ systems by using an exact probabilistic approach and a Gaussian approximation. We showed that the Gaussian approximation of the pdf of the collision-induced time shift is inaccurate because the number of pulse collisions in the system is finite, and hence the result of the central limit theorem holds only approximately. This leads to a large error in the tails of the pdf. However, when another source of randomness, in particular noise from optical amplifiers is included, the difference in the exact and Gaussian approaches is reduced because of noise averaging. Therefore calculation of the BER in the presence of both collision-induced timing jitter and amplifier noise can be carried out with an acceptable accuracy using the Gaussian approximation of the time shift pdf in some cases. From a practical standpoint, if FEC techniques are used, the accuracy of the BER calculation becomes significant when approaching values of 10^{-4} to 10^{-3}, since there is usually a sharp BER threshold above which FEC ceases to function. As the calculation of the time-shift pdf using the characteristic function method is not computationally expensive, we believe that it should always be used instead of the Gaussian approximation.

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