SPATIAL WEIGHTED SPARSE REGRESSION FOR HYPERSPECTRAL IMAGE UNMIXING

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ABSTRACT

Sparse unmixing of hyperspectral data is an important technique which aims at estimating the fractional abundances of endmembers (pure spectral components). It is well known that enforcing sparseness becomes a necessary process in sparse unmixing methods. To better exploit the sparsity in hyperspectral imagery, a double reweighted sparse unmixing algorithm has been proposed. However, it focusses on analyzing the hyperspectral data without fully incorporating the spatial information. To address this limitation, a spatial weighted sparse unmixing (SWSU) algorithm is proposed in this paper, which can take full advantage of the spatial information and further enhance the sparsity of the abundances. This is done by incorporating local neighborhood weights into the double reweighted sparse unmixing formulation. Experimental results on simulated hyperspectral data sets illustrate the good potential of the spatial weighted strategy for sparse unmixing introduced in this paper, which can greatly improve abundance estimation results.

Index Terms— Hyperspectral imaging, sparse unmixing, spatially weighted unmixing.

1. INTRODUCTION

Due to insufficient spatial resolution and spatial complexity, pixels in remotely sensed hyperspectral images are likely to be formed by a mixture of pure spectral constituents (*endmembers*) rather than a single substance [1]. Spectral unmixing, which estimates the fractional abundances of the pure spectral signatures or endmembers in each mixed pixel, was proposed to deal with the problem of spectral mixing and effectively identifies the components of the mixed spectra in each pixel [2].

For the past few years, linear spectral unmixing has been one of the most active research lines to deal with mixed pixels. According to whether a spectral library is available or not, researchers have developed unsupervised [3] and semi-supervised unmixing [4] algorithms. However, such unsupervised algorithms based on extracting the endmembers directly from the scene have faced difficulties, mainly related with the unavailability of pure signatures in the image data. In addition, endmember generation algorithms could obtain virtual endmembers with no physical meaning. Furthermore, there is a need to estimate the number of endmembers in a given scene, which is also a difficult problem.

Recently, as spectral libraries have become widely available, sparse unmixing [4] (as a semi-supervised approach in which mixed pixels are expressed in the form of combinations of a number of pure spectral signatures from a large spectral library) is now able to handle the drawbacks introduced by such *virtual* endmembers and the unavailability of pure pixels. The sparse unmixing approach exhibits significant advantages over unsupervised approaches, as it does not need to extract endmembers from the hyperspectral data or estimate the number of the endmembers. Furthermore, sparse unmixing techniques can estimate the abundances more accurately.

As the number of actual endmembers present in the hyperspectral scene is usually much smaller than the number of spectra in the library, the fractional abundances are often quite sparse. The sparse unmixing algorithm via variable splitting and augmented Lagrangian (SUnSAL) [4] was proposed and has been successfully applied for spectral unmixing purposes. Despite the wide use of the SUnSAL algorithm, the sparsity property of the ℓ_1 regularizer and its influence on unmixing performance has not been thoroughly investigated [5]. To enhance the sparsity of the solution and improve the unmixing quality, inspired by the success of weighted ℓ_1 minimization [6] in sparse signal recovery, the double reweighted s-

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parse unmixing algorithm (DRSU) [7] has been proposed and shown to exhibit significant advantages over traditional sparse unmixing approaches. Although the double reweighted sparse unmixing technique has shown obvious benefits, it just treats the remote sensing image as a group of digital signals, without fully considering the spatial arrangement that the pixels in the image possess. As the research into sparse unmixing has progressed, a recent trend is to exploit the spatial-contextual information [8, 9]. The sparse unmixing via variable splitting augmented lagrangian and total variation (SUnSAL-TV) [10] was proposed as a representative algorithm which utilizes the spatial information in a first-order pixel neighborhood system. However, it may lead to over-smoothness and blurred boundaries, leading to wrong estimations.

In this paper, to enhance the sparsity of the abundance and fully consider the spatial correlation between the abundance coefficients in the image, a spatial weighted sparse unmixing (SWSU) approach is proposed. One weight is used to enhance the sparsity of the fractional abundance, according to the fact that the endmembers are sparse in the library [7]. The other weight is used to mine the spatial correlation of abundance coefficients in the image. In SWSU, a local means method incorporates the local neighborhood weights into the double reweighted sparse unmixing formulation. The local means method estimates the abundance value of the current pixel as an average of the values of all the pixels whose neighborhood is similar to the current one according to the corresponding weights. This approach can effectively preserve the spatial correlation among image features in the image and extract the spatial information in a certain size of a sliding window. Based on this new idea, as well as on the advantages of the double reweighted sparse unmixing algorithm, the proposed SWSU algorithm can further enhance the sparsity of the solution and greatly improve unmixing results.

2. RELATED WORK

2.1. Sparse Unmixing

Let $\mathbf{Y} = [\mathbf{y}_1, \ldots, \mathbf{y}_n] \in \mathbb{R}^{d \times n}$ denotes a hyperspectral image with n being the number of pixel vectors in $\mathbf{Y}$ and d being the number of bands. Let $\mathbf{A} \in \mathbb{R}^{d \times m}$ be a large spectral library, where m is the number of spectral signatures in $\mathbf{A}$, and $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_n]$ denotes the abundance maps corresponding to library $\mathbf{A}$ for the observed data $\mathbf{Y}$. With the aforementioned definitions in mind, sparse unmixing finds a linear combination of endmembers for $\mathbf{Y}$ from the spectral library $\mathbf{A}$:

$$\mathbf{Y} = \mathbf{AX} + \mathbf{N} \quad \text{s.t.:} \quad \mathbf{X} \geq 0, \tag{1}$$

where $\mathbf{N} \in \mathbb{R}^{d \times n}$ is the error, $\mathbf{X} \geq 0$ is the so-called nonnegativity constraint (ANC). It should be noted that we explicitly enforce the ANC constraint without the sum-to-one constraint (ASC) constraint, due to some criticisms about the ASC in the literature [4].

As the number of endmembers involved in a mixed pixel is usually very small when compared with the size of the spectral library, the vector of fractional abundances $\mathbf{X}$ is sparse. With these considerations in mind, the unmixing problem can be formulated as an $\ell_2 - \ell_0$ optimization problem,

$$\min_{\mathbf{X}} \quad \frac{1}{2}||\mathbf{AX} - \mathbf{Y}||_F^2 + \lambda||\mathbf{X}||_0 \quad \text{s.t:} \quad \mathbf{X} \geq 0, \tag{2}$$

where $\|\cdot\|_F$ is the Frobenius norm, λ is a regularization parameter. Problem (2) is nonconvex and difficult to solve [11, 12]. The sparse unmixing algorithm via variable splitting and augmented Lagrangian (SUnSAL) alternatively uses the $\ell_2 - \ell_1$ norm to replace the $\ell_2 - \ell_0$ norm and solve the unmixing problem as follows [4]:

$$\min_{\mathbf{X}} \quad \frac{1}{2}||\mathbf{AX} - \mathbf{Y}||_F^2 + \lambda||\mathbf{X}||_1 \quad \text{s.t.:} \quad \mathbf{X} \geq 0, \tag{3}$$

where $||\mathbf{X}||_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} |x_{ij}|$ and x_{ij} is the j-th value of $\mathbf{x}_i$. SUnSAL solves the optimization problem in (3) efficiently using the alternating direction of multipliers (ADMM) method [13]. However, the sparsity property of SUnSAL algorithm and its influence on the unmixing performance have not been thoroughly investigated.

2.2. Double Reweighted Sparse Unmixing

To enhance the sparsity of the solution and improve the unmixing quality, inspired by the success of weighted ℓ_1 minimization [6] in sparse signal recovery, the double reweighted sparse unmixing (DRSU) [7] was proposed as follows:

$$\min_{\mathbf{X}} \quad \frac{1}{2}||\mathbf{AX} - \mathbf{Y}||_F^2 + \lambda||\boldsymbol{\omega}_2 \odot (\boldsymbol{\omega}_1 \mathbf{X})||_{1,1} \quad \text{s.t:} \quad \mathbf{X} \geq 0, \tag{4}$$

where $\boldsymbol{\omega}_1 \in \Re^{m \times m}$ is a diagonal matrix, of the following form:

$$\boldsymbol{\omega}_1^{t+1} = diag(\frac{1}{||\mathbf{X}^t(1,:)||_2 + \varepsilon}, \ldots, \frac{1}{||\mathbf{X}^t(m,:)||_2 + \varepsilon}) \tag{5}$$

where $\mathbf{X}^t(i,:)$ is the ith row in the estimated abundance of the tth iteration, ε is a small positive value, and the operator $\odot$ denotes the element-wise multiplication of two variables. The weight $\boldsymbol{\omega}_1$ works similarly as in [6] to promote the sparsity of the rows in the abundance matrix largely. In addition, the weight $\boldsymbol{\omega}_2 = [\boldsymbol{\omega}_{2,1}, \ldots, \boldsymbol{\omega}_{2,n}] \in \Re^{m \times n}$ is defined as follows:

$$\boldsymbol{\omega}_{2,i}^{t+1} = \frac{1}{\widehat{\mathbf{x}}_i^t + \varepsilon}, \tag{6}$$

where $\widehat{\mathbf{x}}_i^t$ is the entry in the estimated abundance of the tth iteration. In DRSU, $\boldsymbol{\omega}_1$ enhances the sparsity of nonzero rows corresponding to the true endmembers in the estimated abundance, while the sparsity of the nonzero entries in the nonzero rows is promoted by $\boldsymbol{\omega}_2$ [7].

This formulation generally suggests that large weights could be used to discourage nonzero entries in the recovered signal, while small weights could be used to encourage nonzero entries. The method introduces several steps of weighted ℓ_1 optimization, and uses the value of the current solution to revise the weights for the next iteration, which makes the sparsity of fractional abundances of mixed pixel be represented better. Compared to the classic sparse unmixing algorithm, the double reweighted sparse unmixing technique has shown obvious benefits, but it does not take into account the spatial information.

3. PROPOSED METHOD

3.1. Local Neighborhood Weights

Let $\mathbf{x}_i \in \mathbf{X}$ represent the fraction of the i-th pixel in the abundance image. Let $\mathbf{x}_j$ be a neighboring pixel of $\mathbf{x}_i$ for $j \in \mathcal{N}(i)$, where $\mathcal{N}(i)$ is the neighboring set of i. Let $\boldsymbol{\epsilon}_{ij}$ be the spatial weight for pixel $\mathbf{x}_i$ and $\mathbf{x}_j$, which is computed as follows:

$$\boldsymbol{\epsilon}_{ij} = \frac{1}{\alpha}, \;\; j \in \mathcal{N}(i), \tag{7}$$

where α is a parameter controlling the degree of spatial influence. Let (e, f) and (g, h) be the spatial coordinates of $\mathbf{x}_i$ and $\mathbf{x}_j$. Then, the notations $\mathbf{x}(e, f)$ and $\mathbf{x}(g, h)$ refer to $\mathbf{x}_i$ and $\mathbf{x}_j$, respectively. Under this setting, we can compute α as

$$\alpha = \sqrt{(e-g)^2 + (f-h)^2}.$$

It is observable that, if $\mathbf{x}_i$ and $\mathbf{x}_j$ are closer, $\boldsymbol{\epsilon}_{ij}$ is bigger. Therefore, the spatial significance in this case is more relevant than the other way around. In this work, two neighboring systems, including 3×3 and 5×5 windows neighborhoods are considered.

3.2. Spatial Weighted Sparse Unmixing

To take full advantage of the spatial information, a new sparse unmixing method called spatial weighted sparse unmixing (SWSU) algorithm is proposed for hyperspectral imagery. Our optimization problem can be defined as follows:

$$\min_{\mathbf{X}} \quad \frac{1}{2}||\mathbf{AX} - \mathbf{Y}||_F^2 + \lambda||\widehat{\boldsymbol{\omega}}_2 \odot (\boldsymbol{\omega}_1 \mathbf{X})||_{1,1} \quad \text{s.t:} \quad \mathbf{X} \geq 0, \tag{8}$$

where $\boldsymbol{\omega}_1$ denotes the spectral sparsity weights given in (5), and $\widehat{\boldsymbol{\omega}}_2 = [\widehat{\boldsymbol{\omega}}_{2,1}, \ldots, \widehat{\boldsymbol{\omega}}_{2,n}] \in \Re^{m \times n}$ denotes the spatial sparsity controlling weights, which are given by:

$$\widehat{\boldsymbol{\omega}}_{2,i}^{t+1} = \frac{1}{\sum_{j \in \mathcal{N}(i)} \boldsymbol{\epsilon}_{ij} \widehat{\mathbf{x}}_j^t / \sum_{j \in \mathcal{N}(i)} \boldsymbol{\epsilon}_{ij} + \varepsilon}, \tag{9}$$

where $\widehat{\mathbf{x}}_j^t$ for $j \in \mathcal{N}(i)$ is the entry in the estimated abundance of the tth iteration. This approach plugs spatial information into the model to enforce sparsity.

4. EXPERIMENTAL RESULTS

The library that we use in our synthetic image experiments is a dictionary of minerals extracted from the United States Geological Survey (USGS) library. The library $\mathbf{A}$ contains $m = 240$ materials with $L = 224$ bands. We have simulated a 100×100-pixel synthetic datacube by randomly choosing nine signatures from $\mathbf{A}$. The fractional abundances are piecewise smooth, i.e., they are smooth with sharp transitions, especially in the boundaries of image objects (a detailed description of the simulated data cube is available from [10]). After generating the datacube, it was contaminated with i.i.d. Gaussian noise, using three levels of the signal-to-noise ratio (SNR): 30, 40 and 50dB.

For quantitative analysis, the signal-to-reconstruction error (SRE, measured in dB) is used to evaluate the unmixing accuracy. Let $\widehat{\mathbf{x}}$ be the estimated abundance, and $\mathbf{x}$ be the true abundance. The SRE(dB) can be computed as follows:

$$\text{SRE(dB)} = 10 \cdot \log_{10}(E(||\mathbf{x}||_2^2)/E(||\mathbf{x} - \widehat{\mathbf{x}}||_2^2)), \tag{10}$$

where $E(\cdot)$ denotes the expectation function. Furthermore, we use another indicator, i.e., the probability of success p_s, which is an estimate of the probability that the relative error power be smaller than a certain threshold. It is formally defined as follows: $p_s \equiv P(\|\widehat{\mathbf{x}} - \mathbf{x}\|^2/\|\mathbf{x}\|^2 \leq threshold)$. In our case, the estimation result is considered successful when $\|\widehat{\mathbf{x}} - \mathbf{x}\|^2/\|\mathbf{x}\|^2 \leq 3.16$ (5 dB). This threshold was demonstrated to be appropriate in [4]. The larger the SRE(dB) and p_s, the more accurate the unmixing. Furthermore, the number of elements in $\widehat{\mathbf{x}}$ that are greater than 0.005 are counted, and their proportion with regards to all elements of the abundance matrix is defined as the *sparsity*.

Table 1 shows the SRE(dB), p_s and *sparsity* results achieved by the different tested algorithms under different SNR levels. For all the tested algorithms, the input parameters have been carefully tuned for optimal performance. From Table 1, we can see that the proposed SWSU algorithms (where SWSU-W1 and SWSU-W2 respectively represent the 3×3 or 5×5 pixel neighborhoods adopted for spatial weighted sparse unmixing) obtain better SRE(dB) results than other algorithms in all cases. The p_s obtained by the proposed approaches are also much better than those obtained by other algorithms in the case of low SNR, which reveals that the inclusion of spatial information leads to high robustness. In addition, the proposed approaches can result in a substantial *sparsity* improvement and can achieve sparser results than the other algorithms. Based on this, we can conclude that the inclusion of a spatial weighted strategy offers the potential to improve unmixing performance in three different analysis scenarios. Finally, the results obtained by SWSU-W1 and SWSU-W2 are very similar, which indicates that different window sizes have little effect on the final results.

Table 1. SRE(dB), p_s and *sparsity* scores achieved after applying different unmixing methods to the simulated data sets (the optimal parameters for which the reported values were achieved are indicated in the parentheses).

Algorithm	**SNR=30dB**			**SNR=40dB**			**SNR=50dB**		
	SRE(dB)	p_s	*sparsity*	SRE(dB)	p_s	*sparsity*	SRE(dB)	p_s	*sparsity*
SUnSAL	8.4373	0.7946	0.0497	15.1721	0.9886	0.0426	23.0894	1	0.0257
	($\lambda = 2e$-2)			($\lambda = 5e$-3)			($\lambda = 1e$-3)		
SUnSAL-TV	11.4304	0.9470	0.0552	17.7695	0.9998	0.0341	26.1655	1	0.0178
	($\lambda = 1e$-2; $\lambda_{TV} = 4e$-3)			($\lambda = 5e$-3; $\lambda = 1e$-3)			($\lambda = 2e$-3; $\lambda = 2e$-4)		
DRSU	14.9876	0.9745	0.0249	29.6861	1	0.0137	41.1967	1	0.0120
	($\lambda = 3e$-3)			($\lambda = 1e$-3)			($\lambda = 6e$-4)		
SWSU-W1	19.9548	0.9978	0.0200	31.9039	1	0.0126	41.3384	1	0.0120
	($\lambda = 5e$-3)			($\lambda = 3e$-3)			($\lambda = 5e$-4)		
SWSU-W2	19.8436	0.9995	0.0212	31.6927	1	0.0128	41.3036	1	0.0120
	($\lambda = 3e$-3)			($\lambda = 3e$-3)			($\lambda = 5e$-4)		

5. CONCLUSIONS AND FUTURE WORK

In this work, we have revisited the sparse unmixing formulation and incorporated local neighborhood weights into the double reweighted sparse unmixing to account for spatial information. The newly proposed SWSU algorithm can improve the sparsity of the abundance by fully considering the spatial correlation between the abundance coefficients in the image. Our experiments with simulated hyperspectral data reveal that the SWSU algorithm consistently achieves good spectral unmixing performance in comparison with other algorithms. Future work will be focused on conducting additional experiments with real hyperspectral images.

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