MATH 251 (Spring 2004) Exam 3, April 28th

No calculators, books or notes!
Show all work and give **complete explanations** for all your answers.
This is a 65 minute exam. It is worth a total of 75 points.

(1) [14 pts] Set up integrals of the form

$$\int_{t=a}^{t=b} h(t)\,dt$$

that are equal to the following integrals, but do NOT evaluate the integrals you set up.

(a) $\int_C \ln(x+y)ds$, where C is the curve which is arc of the parabola $y = x^2$ from $(1,1)$ to $(3,9)$.

(b) $\int_C \mathbf{F}\cdot d\mathbf{r}$, where C is the curve parametrized by $r(t) = (1+2t, 3+4t^2)$ for $0 < t < 2$ and $\mathbf{F}(x,y) = x^2\mathbf{i} + \sin(y)\mathbf{j}$.

(2) [13 pts]
(a) Calculate $\iint_D y\,dA$, where D is the region in the first quadrant of the xy-plane that lies above the hyperbola $xy = 1$, above the line $y = x$ and below the line $y = 2$.

(b) Find a, b, $f_1(x)$ and $f_2(x)$ so that

$$\int_{y=0}^{y=1} \int_{x=3y}^{x=3} e^{x^2}\, dx\, dy = \int_{x=a}^{x=b} \int_{y=f_1(x)}^{y=f_2(x)} e^{x^2}\, dy\, dx$$

(3) [14 pts] Consider the two vector fields

$$\begin{aligned}\mathbf{F}_1(x,y) &= (2xy - 2y^2\sin x)\mathbf{i} \ + \ (x^2 + 4y\cos x)\mathbf{j} \\ \mathbf{F}_2(x,y) &= (2xy^2 - 2y\sin x)\mathbf{i} \ + \ (x^2 + 4y^2\cos x)\mathbf{j}\end{aligned}$$

One of these vector fields is conservative.

(a) Which vector field is conservative and which is not? Why?

(b) For the vector field that is conservative, evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is any curve from $(0,0)$ to $(0,1)$.

(4) [12 pts]
(a) Carefully state Green's Theorem

(b) Use Green's Theorem to evaluate $\int_C x^2y dx - xy^2 dy$, where C is the circle $x^2 + y^2 = 4$ with counter-clockwise orientation.

(5) [12 pts] Use the Method of Largange Multipliers to maximize the function $f(x, y) = xy$ subject to the constraint $4x^2 + y^2 = 16$. [Hint: There are 4 critical points.]

(6) [10 pts] State and prove the Fundamental Theorem of Calculus for Line Integrals.

Pledge: *I have neither given nor received aid on this exam*

Signature: ______________________________