NAME:

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MATH 251 (Fall 2010) Final Exam, Dec 16th

No calculators, books or notes! Show all work and give **complete explanations**. This 120 min exam is worth 100 points.

(1) [8 pts] Calculate the following limits or show they do not exist.

(a) $\lim_{(x,y)\to(0,0)} \frac{x^2-3y^2}{4x^2+7y^2}$

(b) $\lim_{(x,y)\to(0,0)} \frac{2+x+y}{1+x^2+y^2}$

(2) [12 pts]
(a) Calculate the projection of the vector $\mathbf{a} = 3\mathbf{j}$ onto the vector $\mathbf{b} = \mathbf{i} + \mathbf{j}$. Draw a labelled picture that clearly illustrates the relationship between these three vectors.

(b) Find the volume of the parallelipiped determined by the vectors $(1, 2, 3)$, $(0, 1, -4)$, and $(5, 0, 2)$.

(3) [10 pts]

(a) Find an equation of the form $z = ax + by + c$ for the tangent plane to the graph of $z = f(x, y) = 3x^2 + 5y^2$ at the point $(x, y, z) = (1, 2, 23)$.

(b) Calculate $\iint_D y^2 \, dA$, where D is the region in the xy-plane bounded by the curves $y^2 = x$ and $x + y^2 = 8$.

(4) [6 pts] Calculate the length of the curve $\mathbf{r}(t) = (4t, 3\cos t, 3\sin t)$ for $0 \leq t \leq \pi$.

(5) [8 pts] An *anemometer* is an instrument that measures wind speed. Suppose that $\mathbf{F}(x, y) = y\mathbf{i} + 2x\mathbf{j}$ is the velocity vector field of air moving across the xy-plane. Suppose an ant that is carrying an anemometer is at the point $\mathbf{p} = (-1, 4)$ and is walking with velocity $\mathbf{v} = (2, 3)$. Is the wind speed measured by the anemometer increasing or decreasing?

(6) [12 pts] Sketch the following.

(a) The surface $\theta = \frac{-\pi}{4}$

(b) The surface $\phi = \frac{5\pi}{6}$ for $0 \leq \theta \leq 2\pi$ and $1 \leq \rho \leq 2$.

(c) The solid region $0 \leq \theta \leq \frac{\pi}{2}$, $r^2 \leq z \leq 2$.

(7) [12 pts] Find the absolute maximum and minimum of the function $f(x, y) = 2x^2 + y^2 - 2xy - 2x$ on the rectangular region bounded by the lines $x = 0$, $y = 0$, $x = 2$, and $y = 3$.

(8) [12 pts]
(a) Carefully state the Divergence Theorem. You may find it helpful to draw a picture and refer to it in your written explanation.

(b) Let S be the surface $x^2 + y^2 + z^2 = 4$ with the outward orientation, and let $\mathbf{F}$ be the vector field $\mathbf{F} = xz^2\mathbf{i} + \sin(z)\mathbf{j} + xy\mathbf{k}$. Calculate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

(9) [10 pts] Find the volume of the solid region bounded by the surfaces $x^2 + z^2 = 1$, $2y + z = 8$, and $x + y = 1$.

(10) [10 pts] Let S be the surface that is the portion of the paraboloid $y = x^2 + z^2$ with $0 \leq y \leq 4$. We choose the unit normal $\mathbf{n}$ on S to be the one with $\mathbf{n} \cdot \mathbf{j} > 0$. Let $\mathbf{F}(x, y, z) = x\mathbf{i} + z\mathbf{k}$. Calculate $\iint_S \mathbf{F} \cdot d\mathbf{S}$. [Hint: Define parameters for S in terms of polar coordinates in the xz-plane.]

Pledge: *I have neither given nor received aid on this exam*

Signature: ________________________________